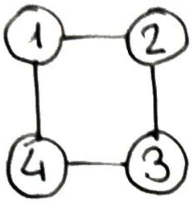
Antenne parabolique

$\boxed{1}$



$\boxed{2}$ Loi Entrée-Sortie : $\theta = f(\lambda)$

Fermeture géométrique $\vec{OA} = \vec{OB} + \vec{BA}$

$$\Rightarrow \; c\vec{x_2} = b\vec{x_1} - a\vec{y_1} + \lambda\vec{y_4} \qquad \begin{cases} \vec{x_2} = \cos\theta\,\vec{x_1} + \sin\theta\,\vec{y_1} \\ \vec{y_4} = \cos\beta\,\vec{y_1} - \sin\beta\,\vec{x_1} \end{cases}$$

$$\Rightarrow \begin{cases} \text{projection}/\vec{x_1} : \; c\cdot\cos\theta = b - \lambda\sin\beta \\ \text{projection}/\vec{y_1} . \; c\cdot\sin\theta = -a + \lambda\cos\beta \end{cases}$$

$$\begin{cases} \lambda\sin\beta = b - c\cdot\cos\theta \\ \lambda\cos\beta = a + c\cdot\sin\theta \end{cases}$$

$$\Rightarrow \lambda^2 = (b - c\cdot\cos\theta)^2 + (a + c\cdot\sin\theta)^2 \qquad (1)$$

D'où : $\lambda = \sqrt{(b - c\cos\theta)^2 + (a + c\cdot\sin\theta)^2}$

$\boxed{3}$

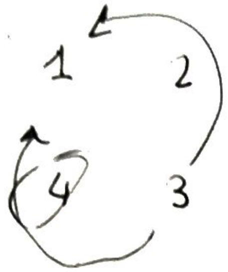
$$(1) : \quad \lambda^2 = b^2 + a^2 + c^2 - 2bc\cos\theta + 2ac\sin\theta$$

$$\Big\downarrow \, d/dt$$

$$2\lambda\dot{\lambda} = 2bc\dot{\theta}\sin\theta + 2ac\dot{\theta}\cos\theta$$

d'où : $$\dot{\theta} = \frac{\sqrt{(b - c\cdot\cos\theta)^2 + (a + c\cdot\sin\theta)^2}}{c(b\cdot\sin\theta + a\cdot\cos\theta)}\dot{\lambda}$$

$\boxed{4}$ Vitesse : $\vec{V}(A \in 3/1)$



$$\vec{V}(A \in 3/1) = \vec{V}(A \in 3/4) + \vec{V}(A \in 4/1)$$

avec : $\vec{V}(A \in 3/4) = \dot{\lambda}\,\vec{y_4}$

et : $\vec{V}(A \in 4/1) = \underbrace{\vec{V}(B \in 4/1)}_{\vec{0}} + \vec{\Omega}(4/1) \wedge \vec{BA}$

$$\Rightarrow \vec{V}(A \in 4/1) = \dot{\beta}\,\vec{z_1} \wedge \lambda\,\vec{y_4} = \lambda\dot{\beta}\,\vec{x_4}$$

d'où : $\vec{V}(A \in 3/4) = \dot{\lambda}\,\vec{y_4} - \lambda\dot{\beta}\,\vec{x_4}$

$\boxed{5}$ Vitesses $\vec{V}(A \in 2/1)$

$$\vec{V}(A \in 2/1) = \vec{V}(O \in 2/1) + \vec{\Omega}(2/1) \wedge \vec{OA}$$

$$\Rightarrow \vec{V}(A \in 2/1) = \dot{\theta}\,\vec{z_2} \wedge c\,\vec{x_2} = c\dot{\theta}\,\vec{y_2}$$

On compose les vitesses en A, soit : $\vec{V}(A \in 3/1) = \vec{V}(A \in 3/2) + \vec{V}(A \in 2/1)$

d'où $\dot{\lambda}\,\vec{y_4} - \lambda\dot{\beta}\,\vec{x_4} = c\dot{\theta}\,\vec{y_2}$ (2)

Produit scalaire de (2) par $\vec{y_4}$ pour éliminer $\dot{\beta}$, soit :

$$\dot{\lambda} = c\dot{\theta}\,\vec{y_2} \wedge \vec{y_4} = c\dot{\theta}\cos(\theta - \beta)$$