**ES 012**

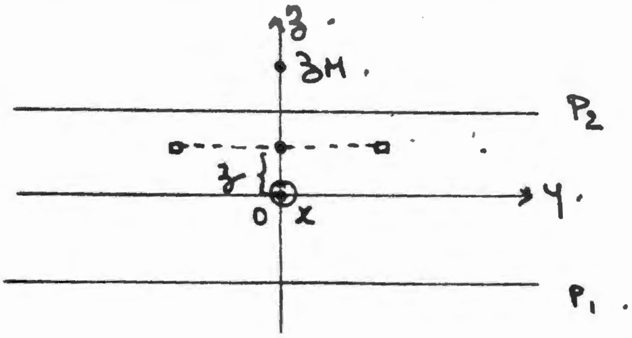
$$V = \frac{2q}{4\pi\varepsilon_0 r} - \frac{q}{4\pi\varepsilon_0 r_1} - \frac{q}{4\pi\varepsilon_0 r_2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta + \left(\frac{a}{r}\right)^2 \frac{3\cos^2\theta - 1}{2} + \cdots\right)$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta + \left(\frac{a}{r}\right)^2 \frac{3\cos^2\theta - 1}{2} + \cdots\right)$$

$$\boxed{V = \frac{qa^2(1-3\cos^2\theta)}{4\pi\varepsilon_0 r^3}}$$

**ES 014**




anneau : $\rho$ $(d\rho, dz)$

charge $d^2q = \beta \times 2\pi\rho\, d\rho\, dz$

$$\Rightarrow d^2\vec{E}_M = \frac{d^2q}{4\pi\varepsilon_0} \frac{(z_M - z)}{\left(\rho^2 + (z_M - z)^2\right)^{3/2}} \vec{k}$$

~~Si $|z_M|<a$ : les champs créés par 2 anneaux sym / à (xy) créent en M 2 champs~~

$\cancel{\Rightarrow \vec{E}_M =}$

$$d\vec{E}_M = \int_0^\infty d^2\vec{E}_M = \frac{\beta 2\pi\,(z_M-z)\,dz}{4\pi\varepsilon_0}\int_0^\infty \frac{\rho\, d\rho}{\left(\rho^2+(z_M-z)^2\right)^{3/2}}\ \vec{u}_z .$$

$$d\vec{E}_M = \frac{\beta}{2\varepsilon_0}(z_M-z)\frac{1}{|z_M-z|} = \pm\frac{\beta dz}{2\varepsilon_0}$$ champ uniforme créé par la tranche dz entière

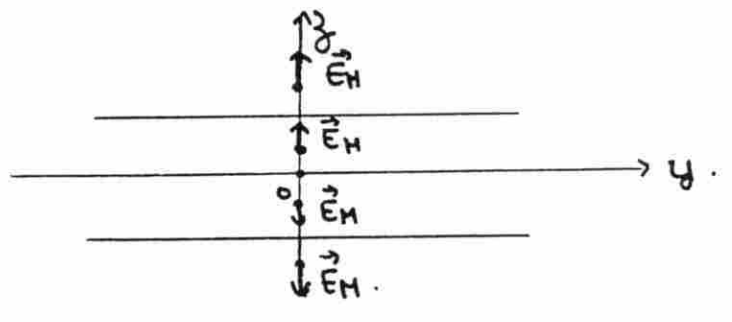
* Si $|z_M|<a$ : les champs créés par 2 tranches de part et d'autre de M sont opposées en direction.

$$\vec{u}_z\cdot\vec{E}_M = \int_{-a}^{z} +\frac{\beta dz}{2\varepsilon_0} + \int_{z}^{a} -\frac{\beta dz}{2\varepsilon_0} = \frac{\beta}{2\varepsilon_0}(z+a) - \frac{\beta}{2\varepsilon_0}(a-z)$$

$$\boxed{\vec{E}_M = \frac{\beta z_M}{\varepsilon_0}\vec{u}_z}$$

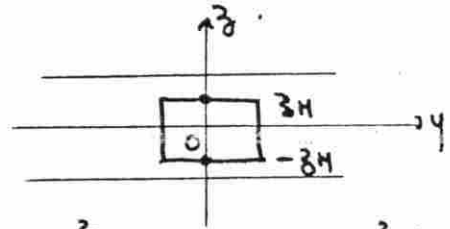
* Si $|z_M|>a$ : les champs ont même sens pour toutes les tranches

$$\vec{E}_M = \int_{-a}^{a} \pm\frac{\beta dz}{2\varepsilon_0}\vec{u}_z = \boxed{\pm\frac{\beta a}{\varepsilon_0}\vec{u}_z}$$ $\begin{cases} \oplus \text{ si } z_M>a \\ \ominus \text{ si } z_M<-a. \end{cases}$




20 Gauss : sym. $\Rightarrow \vec{E} \parallel \vec{u}_z$ et dépend seul$^t$ de z.

* si $|z_M|\le a$ :

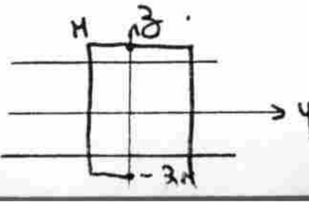



cylindre de hauteur $2|z_M|$ de rayon $\rho$ qcq.

$$\phi = \frac{1}{\varepsilon_0}\beta\pi\rho^2\times 2z_M = \pi\rho^2\left(E(z_M)-E(-z_M)\right) \quad \text{si } z_M>0.$$
$$= 2\pi\rho^2 E(z_M) \quad \text{car } E(z_M) = -E(-z_M).$$

$$\Rightarrow \boxed{E(z_M) = \frac{\beta z_M}{\varepsilon_0}.}$$

* si $|z_M|\ge a$. $(z_M>0)$




cylindre de hauteur $2z_M$ de rayon $\rho$.

$$\phi = \frac{1}{\varepsilon_0} \beta \pi \rho^2 2a = \rho^2 \pi \left( E(z_M) - E(-z_M) \right) = 2\rho^2 \pi \, E(z_M)$$

$$\Rightarrow \boxed{E(z_M) = \frac{\beta a}{\varepsilon_0}} \quad \text{si } z_M > a.$$

$$\phi = +\frac{1}{\varepsilon_0} \beta \pi \rho^2 2a = \rho^2 \pi \left( -E(z_M) + E(-z_M) \right) \Rightarrow \boxed{E(z_M) = -\frac{\beta a}{\varepsilon_0}} \quad \text{si } z_M < -a.$$

3°) Potentiel $\quad E_M = -\dfrac{dV_M}{dz}$.

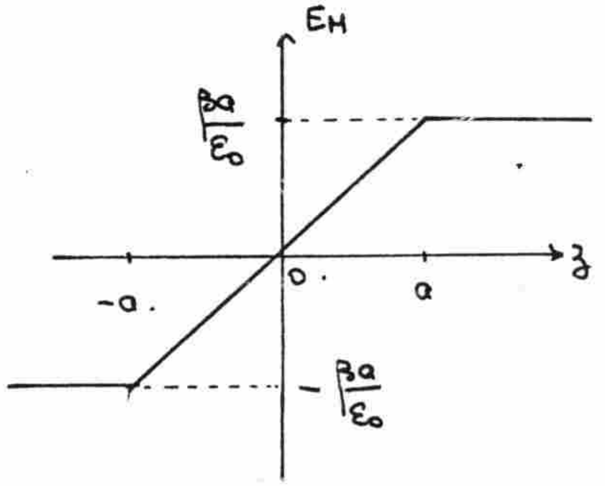
- si $|z_M| \leq a$ : $\quad V_M = -\frac{1}{2} \frac{\beta z_M^2}{\varepsilon_0} + C^{te} \quad V(0) = 0 \Rightarrow \boxed{V_M = -\frac{1}{2} \frac{\beta z_M^2}{\varepsilon_0}}$.

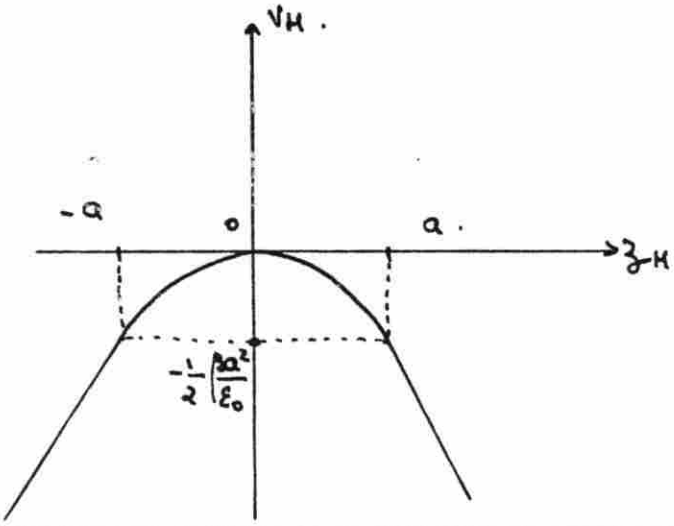
- si $|z_M| \geq a$ : $\quad V_M = \pm \frac{\beta a}{\varepsilon_0} z_M + C^{te}$

pour $z_M \leq -a$ : $\quad V_M(-a) = -\frac{1}{2} \frac{\beta a^2}{\varepsilon_0} = -\frac{\beta a^2}{\varepsilon_0} + C^{te} \Rightarrow C^{te} = \frac{1}{2} \frac{\beta a^2}{\varepsilon_0}$.

alors $\boxed{V_M = \frac{1}{2} \frac{\beta a^2}{\varepsilon_0} + \frac{\beta a}{\varepsilon_0} z_M}$.

$z_M \geq a$ : $\quad V_M(a) = -\frac{1}{2} \frac{\beta a^2}{\varepsilon_0} = -\frac{\beta a^2}{\varepsilon_0} + C^{te} \Rightarrow \boxed{V_M = +\frac{1}{2} \frac{\beta a^2}{\varepsilon_0} - \frac{\beta a}{\varepsilon_0} z_M}$

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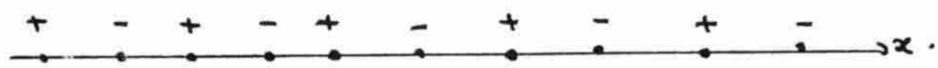
$\boxed{\text{ES } 105}$




$$W = \frac{1}{2} \sum_{i,j} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} = \frac{1}{2} \sum_i q_i V_i$$

calculons ~~$V_0$ créé par la dist$^{ou}$ sur une charge $q_i = +e$.~~

$$V_0 = \frac{e}{4\pi\varepsilon_0} \left[ -\frac{2}{a} + \frac{2}{2a} - \frac{2}{4a} + \frac{2}{6a} \cdots \right]$$

$$= -\frac{2e}{4\pi\varepsilon_0} \left[ \frac{1}{a} - \frac{1}{2a} + \frac{1}{4a} + \frac{2}{6a} \cdots \right] = \frac{-2e}{4\pi\varepsilon_0} \frac{1}{a} \sum_{n=0}^{N_A} \left( -\frac{1}{2} \right)^n$$

$$V_0 = \frac{-2e}{4\pi\varepsilon_0 a} \frac{(-1/2)^{N_A}}{1 + \frac{1}{2}} \# -\frac{4e}{12\pi\varepsilon_0 a}.$$

$$W = -\frac{1}{2}\, \sigma_A \frac{4e^2}{12\pi\varepsilon_0 a} \Rightarrow \boxed{W = -\frac{\sigma_A e^2}{6\pi\varepsilon_0 a}}.$$

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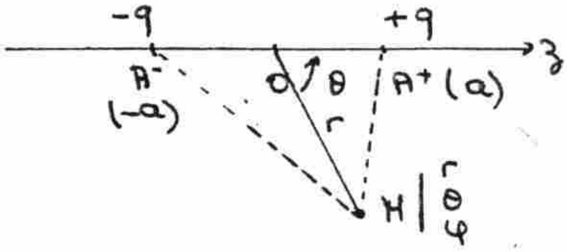
**ES 015**




$$V^+(M) = \frac{q}{4\pi\varepsilon_0}\frac{1}{\|\overrightarrow{A^+M}\|} \qquad V^-(M) = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\|\overrightarrow{A^-M}\|}$$

$$\|\overrightarrow{A^+M}\| = \left(r^2+a^2-2ar\cos\theta\right)^{1/2}$$
$$\|\overrightarrow{A^-M}\| = \left(r^2+a^2+2ar\cos\theta\right)^{1/2}.$$

$$\frac{1}{\|\overrightarrow{A^+M}\|} - \frac{1}{\|\overrightarrow{A^-M}\|} = \frac{1}{r}\left[\left(1-\frac{2a\cos\theta}{r}+\frac{a^2}{r^2}\right)^{-1/2} - \left(1+\frac{2a\cos\theta}{r}+\frac{a^2}{r^2}\right)^{-1/2}\right]$$
$$= \frac{1}{r}\left(+\frac{a\cos\theta}{r}+\frac{a\cos\theta}{r}\right) = +\frac{2a\cos\theta}{r^2}$$

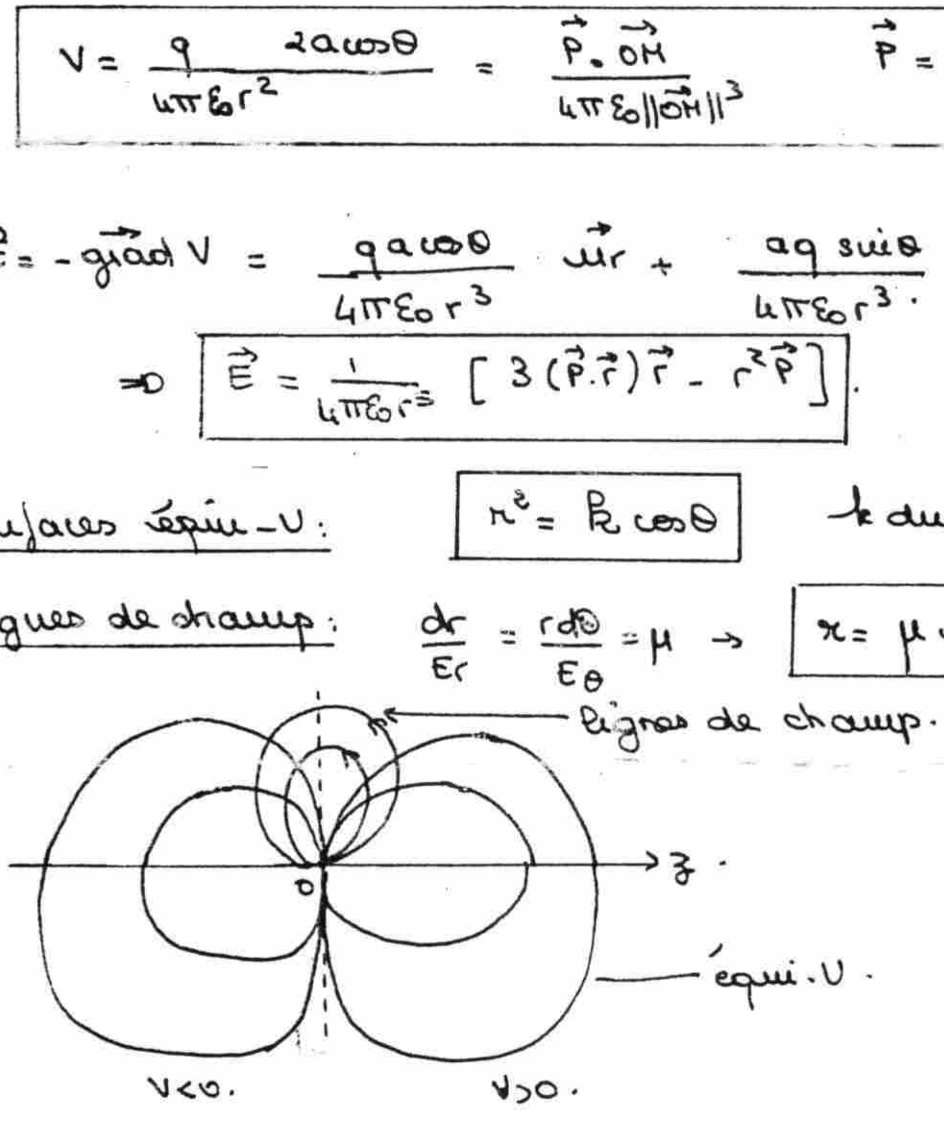
$$\Rightarrow \boxed{V = \frac{q}{4\pi\varepsilon_0 r^2}\,2a\cos\theta = \frac{\vec{P}\cdot\overrightarrow{OM}}{4\pi\varepsilon_0\|\overrightarrow{OM}\|^3} \qquad \vec{P} = 2a\,\vec{u}_z}$$

$$\vec{E} = -\overrightarrow{\text{grad}}\,V = \frac{qa\cos\theta}{4\pi\varepsilon_0 r^3}\,\vec{u}_r + \frac{aq\sin\theta}{4\pi\varepsilon_0 r^3}\,\vec{u}_\theta$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0 r^5}\left[3(\vec{P}\cdot\vec{r})\vec{r} - r^2\vec{P}\right]}.$$

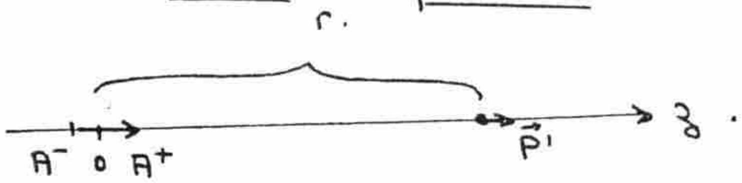
Surfaces équi-V : $\boxed{r^2 = k\cos\theta}$ k du signe de V.

lignes de champ : $\frac{dr}{E_r} = \frac{r\,d\theta}{E_\theta} = \mu \rightarrow \boxed{r = \mu\sin^2\theta}$ cercles.



2°

## $1^{er}$ cas : parallèles.




$$\vec{f}_{p'\to p} = q\vec{E}(r^-) - q\,\vec{E}(r^+) \qquad \begin{cases} r^+ = r-a \\ r^- = r+a \end{cases}$$

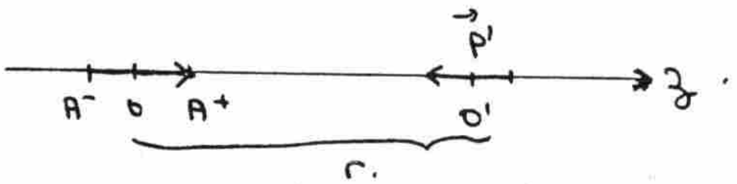
$$\vec{f}_{p'\to p} = \left(q\frac{2p' r_+^2}{4\pi\varepsilon_0 r_+^5} - q\frac{2p' r_-^2}{4\pi\varepsilon_0 r_-^5}\right)\vec{u}_z = q\cdot\frac{2p'}{4\pi\varepsilon_0}\left(\frac{1}{(r-a)^3} - \frac{1}{(r+a)^3}\right)\vec{u}_z$$

$$\# \frac{2p'q}{4\pi\varepsilon_0 r^3}\left(\left(1+\frac{3a}{r}\right) - \left(1-\frac{3a}{r}\right)\right)\vec{u}_z$$

$$\boxed{\vec{f}_{p'\to p} = \frac{3p'qa}{\pi\varepsilon_0 r^3}}\,\vec{u}_z. \qquad \text{soit} \quad \boxed{\vec{f}_{p'\to p} = \frac{3}{2}\frac{p.p'}{\pi\varepsilon_0 r^3}\,\vec{u}_z.}$$

force attractive

## 2° cas : antiparallèles.




$$\vec{f}_{p'\to p} = q\left(\vec{E}(\vec{r}^-) - \vec{E}(\vec{r}^+)\right)$$

$$\vec{E}^+ = \frac{3(\vec{p}'.\vec{O'A^+})\,\vec{O'A^+} - r_+^2\,\vec{p}'}{4\pi\varepsilon_0 r^{+5}} = \frac{-3p' r^{+2} + r^{+2}p'}{4\pi\varepsilon_0 r^{+5}}$$

$$\vec{E}^+ = \frac{-2p'}{4\pi\varepsilon_0 r^{+5}}$$

$$\vec{E}^- = \frac{-2p'}{4\pi\varepsilon_0 r^{-5}}.$$

$$\rightarrow \quad \boxed{\vec{f}_{p'\to p} = -\frac{3}{2}\frac{pp'}{\pi\varepsilon_0 r^3}\,\vec{u}_z}$$

force répulsive

$\boxed{\text{MES 016}}$

1°) $\dfrac{d^2(rV)}{dr^2}=0 \Rightarrow rV = ar+b \Rightarrow V = a+\dfrac{b}{r}$

$V\to 0$ si $r\to\infty$ : $\boxed{V=\dfrac{b}{r}}$ potentiel coulombien

$\vec{E} = \dfrac{b}{r^2}\vec{u}_r$ Gauss $\Rightarrow b = \dfrac{q}{4\pi\varepsilon_0}$. $\Rightarrow \boxed{V=\dfrac{q}{4\pi\varepsilon_0 r}}$

2°) • $\Delta V = -\Lambda$ $\forall r\neq 0 \Rightarrow \dfrac{d^2(rV)}{dr^2} = -\Lambda r$

$\Rightarrow V(r) = \dfrac{1}{6}r^2\Lambda + \alpha + \dfrac{\beta}{r}$ $V(r)\to 0$ si $r\to\infty$
une seule solution convient $\boxed{\Lambda=0}$
alors on retrouve le potentiel coulombien.

• $\dfrac{1}{r}\dfrac{d^2(rV)}{dr^2} = \dfrac{V}{a^2} \Rightarrow \dfrac{d^2(rV)}{dr^2} = \dfrac{rV}{a^2}$

$\Rightarrow rV = \alpha e^{-r/a} + \beta e^{r/a}$ $V\to 0$ si $r\to\infty \Rightarrow \beta=0$.

d'où $\boxed{V(r) = \alpha\dfrac{e^{-r/a}}{r}}$ Potentiel de Yukawa.
Th. de Gauss sur sphère $r\to 0$ : $\alpha = q/4\pi\varepsilon_0$

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$\boxed{\text{ES 019}}$

$\rho(r) = A r^{-n}$

(1°) $Q = 0 = Ze + \int_0^\infty A r^{-n}\, 4\pi r^2 dr = Ze + 4\pi A \underbrace{\int_a^\infty r^{2-n} dr}_{\text{converge si } n>3}$.

alors : $0 = Ze + 4\pi A \dfrac{-1}{-3+n}\left[\dfrac{1}{r^{n-3}}\right]_a^\infty = Ze - \dfrac{4\pi A}{n-3}\left(-\dfrac{1}{a^{n-3}}\right)$.

$\Rightarrow \boxed{A = -\dfrac{Ze\,(n-3)}{4\pi}\, a^{n-3}}$.

(2°) (MG) $\operatorname{div}\vec{E} = \dfrac{\rho}{\varepsilon_0} \Rightarrow \dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2E(r)\right) = -\dfrac{Ze\,(n-3)}{4\pi\varepsilon_0}a^{n-3}\dfrac{1}{r^n}$.

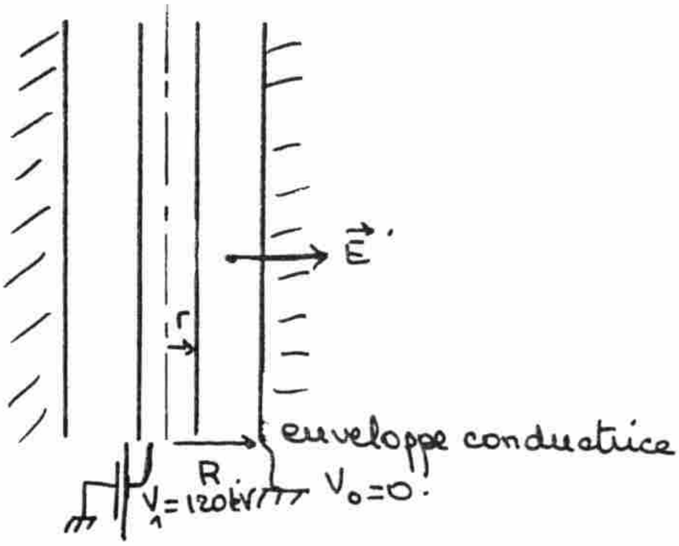
$\Rightarrow \dfrac{d}{dr}\left(r^2E(r)\right) = \dfrac{A}{r^{n-2}} \Rightarrow r^2E(r) = -\dfrac{A}{n-3}\dfrac{1}{r^{n-3}} + B$

$\Rightarrow E(r) = -\dfrac{A}{n-3}\dfrac{1}{r^{n-1}} + \dfrac{B}{r^2}$ on prend $B=0$ ici

$\Rightarrow \boxed{E(r) = \dfrac{Ze\,a^{n-3}}{4\pi\varepsilon_0}\dfrac{1}{r^{n-1}}}$ et $\boxed{V(r) = \dfrac{Ze\,a^{n-3}}{4\pi\varepsilon_0(n-2)}\dfrac{1}{r^{n-2}}}$

(3°) Ici on a : $\rho(r) = K\,V(r)^{3/2} \Rightarrow r^n = r^{3(n-2)/2} \Rightarrow \boxed{n=6}$

$\boxed{V(r) = \dfrac{Ze\,a^3}{16\pi\varepsilon_0 r^4}}$




**1°** <u>Th. de Gauss :</u> $\quad$ <u>$r < \rho < R$ :</u> $\quad E(\rho) = \dfrac{\lambda h}{2\pi h \rho} = A/\rho$.

$$\int_r^R \vec{E} \cdot \vec{dl} = V_1 = A \ln \frac{R}{r} \qquad \text{d'où} \qquad \boxed{E(\rho) = \frac{V_1}{\ln \frac{R}{r}} \frac{1}{\rho}}$$

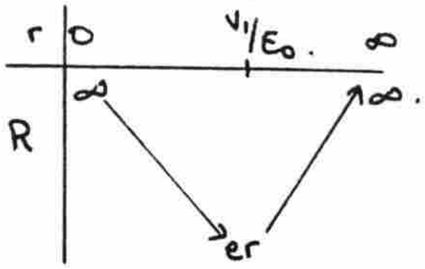
**2°** $E \leq E_0 = 8 \cdot 10^6\ V.m^{-1}$ (champ disruptif).

$$\Rightarrow \frac{V_1}{\left(\ln \frac{R}{r}\right)\rho} \leq \frac{V_1}{\ln \frac{R}{r} \times r} \leq E_0 \Rightarrow \ln \frac{R}{r} \geq \frac{V_1}{E_0 r}$$

$$\Rightarrow \boxed{R \geq R_{min} = r\, e^{V_1/E_0 r}}$$

**3°** $R = r\, e^{V_1/E_0 r} \qquad \dfrac{dR}{dr} = e^{V_1/E_0 r}\left(1 - \dfrac{V_1 r}{E_0 r^2}\right)$

$\Rightarrow$ R minimale pour $\boxed{r = \dfrac{V_1}{E_0}}$




Rmin vaut alors $\boxed{R_0 = e\,r}$

<u>AN :</u> $r = 1{,}5$ cm. $R_0 = 4$ cm.

**4°** <u>Volume d'isolant compris entre les 2 conducteurs :</u>

$$\mathcal{V} = \pi(R^2 - r^2) \times h = \pi h r^2 (x^2 - 1)$$

et on a toujours : $R = r\, e^{V_1/E_0 r} \Rightarrow x = e^{V_1/E_0 r}$

$$\Rightarrow r = \frac{V_1}{E_0 \ln x} \qquad \text{d'où} \qquad \mathcal{V} = \frac{\pi h V^2}{E^2} \frac{x^2 - 1}{(\ln x)^2}.$$

F(x) est minimum si : $\ln x - \left(1 - \dfrac{1}{x^2}\right) = 0. \Rightarrow \boxed{x = 2{,}218}$

ES 020



1°) Par symétrie : $\boxed{\vec{E}(0) = \vec{0}}$

2°) $$\boxed{V(z) = \frac{q}{\pi \varepsilon_0 \left(z^2 + \frac{a^2}{2}\right)^{1/2}}}$$

3°) $\vec{E}(M) \parallel Oz$ pour $M \in Oz$

$$\vec{E}(M) = \frac{4 q \cos\alpha}{4\pi\varepsilon_0 \left(z^2 + \frac{a^2}{2}\right)} \vec{u}_z \quad \text{avec} \quad \cos\alpha = \frac{z}{\left(z^2 + \frac{a^2}{2}\right)^{1/2}}$$

$$\Rightarrow \boxed{\vec{E}(z) = \frac{q z}{\pi \varepsilon_0 \left(z^2 + \frac{a^2}{2}\right)^{3/2}} \vec{u}_z}$$

4°)

| $z$ | $-\infty$ | | $0$ | | $+\infty$ |
|---|---|---|---|---|---|
| $V$ | $0$ | ↗ | $V_0$ | ↘ | $0$ |

$$V_0 = \frac{\sqrt{2}\, q}{\pi \varepsilon_0 a} = V(0)$$

$$\frac{dV}{dz} = -\frac{q}{\pi\varepsilon_0} \frac{z}{\left(z^2 + \frac{a^2}{2}\right)^{3/2}}$$

$$\frac{d^2V}{dz^2} = -\frac{q}{\pi\varepsilon_0}\left[\frac{\left(z^2+\frac{a^2}{2}\right)^{3/2} - 3z^2\left(z^2+\frac{a^2}{2}\right)^{1/2}}{\left(z^2+\frac{a^2}{2}\right)^3}\right] = -\frac{q}{\pi\varepsilon_0} \frac{\frac{a^2}{2} - 2z^2}{\left(z^2+\frac{a^2}{2}\right)^{5/2}}$$

points d'inflexion : $\boxed{z = \pm \frac{a}{2}}$

---

$$\frac{dE}{dz} = \frac{q}{\pi\varepsilon_0} \frac{\frac{a^2}{2} - 2z^2}{\left(z^2 + \frac{a^2}{2}\right)^{5/2}}$$

$$E_0 = E\left(\frac{a}{2}\right) = \frac{q}{2\pi\varepsilon_0 a^2}$$

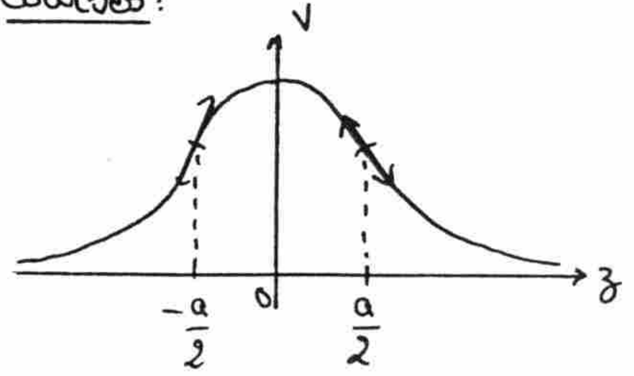
| $z$ | $-\infty$ | | $-\frac{a}{2}$ | | $0$ | | $\frac{a}{2}$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{dE}{dz}$ | | $-$ | $0$ | $+$ | | $+$ | $0$ | $-$ | |
| $E(z)$ | $0$ | ↘ | $-E_0$ | ↗ | $0$ | ↗ | $E_0$ | ↘ | $0$ |

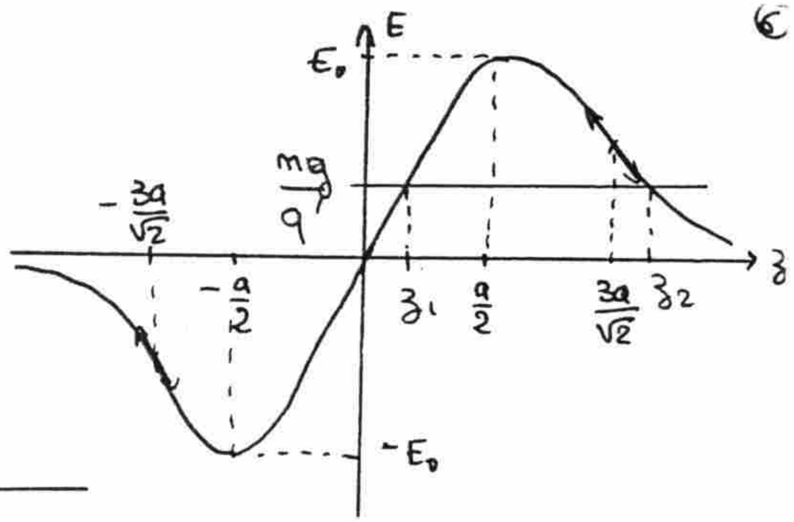
Points d'inflexion : $$\frac{d^2E}{dz^2} = \frac{q}{4\varepsilon_0}\left[\frac{5a^2}{2} \frac{-3z}{(\quad)^{7/2}} + \frac{-4z}{(\quad)^{5/2}} + \frac{10 z^3}{(\quad)^{7/2}}\right]$$

$$\frac{d^2E}{dz^2} = 0 \Leftrightarrow \frac{5a^2}{2} z + 4z\left(\frac{a^2}{2} + z^2\right) - 10 z^3 = 0$$

$$\Leftrightarrow \begin{cases} z = 0 \\ \text{ou} \\ 5a^2 + 4a^2 + 8z^2 - 10z^2 = 0 \end{cases} \Leftrightarrow \boxed{\begin{array}{c} z = 0 \\ \text{et} \\ z = \pm \frac{3a}{\sqrt{2}} \end{array}}$$

Courbes :




4°) Equilibre : $q'E(z) - mg = 0$.

$z_e$ vérifie : $$\frac{z_e}{\left(z_e^2 + \frac{a^2}{2}\right)^{3/2}} = \frac{\pi \varepsilon_0 mg}{qq'}$$

graphiquement : pour déterminer $z_e$, on trace $E(z)$ et la droite $y = \frac{mg}{q'}$.

Il ne peut y avoir équilibre que si $\frac{mg}{q'} < E_0$ pour $q' > 0$

(ou $\frac{mg}{q'} > -E_0$ pour $q' < 0$)

Condition d'existence de la position d'éq :
$$\boxed{|q'| > \frac{mg}{E_0} = \frac{2\pi\varepsilon_0 mga}{q}}$$

Stabilité : $q' > 0$ par ex.

$\Rightarrow$ 2 positions d'éq : $z_1 \in [0; \frac{a}{2}]$ $z_2 > \frac{a}{2}$

- pour $z_1$ : $\left(\frac{dE}{dz}\right) > 0 \Rightarrow \left(\frac{d^2V}{dz^2}\right) < 0$ équilibre stable pour $q' < 0$
  instable pour $q' > 0$.

- pour $z_2$ : $\frac{d^2V}{dz^2} > 0 \Rightarrow$ équilibre instable pour $q' < 0$.
  stable pour $q' > 0$.

5°) Petites oscillations : prenons $q' > 0$.

Eq. stable $z = z_2$. PFD à $q'$ : $m\ddot{\varepsilon} = -mg + q'E(z)$ avec $z = z_2 + \varepsilon$

$$E(z) = E(z_2) + \varepsilon \left(\frac{dE}{dz}\right)_{z_2}$$

$$\Rightarrow \boxed{m\ddot{\varepsilon} - q'\left(\frac{dE}{dz}\right)_{z_2} \varepsilon = 0 . \quad T = 2\pi \sqrt{\frac{-m}{q' \underbrace{\left(\frac{dE}{dz}\right)_{z_2}}_{<0}}}}$$

ES 021 <u>Préliminaire</u> :

à l'équilibre : $\overrightarrow{\text{grad}}\, P = \vec{f}_v = n(r) q \vec{E}(r) = - n(r) q \frac{dV(r)}{dr} \vec{u}_r$

$$\Rightarrow kT\left(\frac{dn}{dr}\right) + q\left(\frac{dV}{dr}\right) n(r) = 0$$

$$\Rightarrow \frac{dn}{n} = -\frac{q}{kT} dV \quad \Rightarrow \quad \ln n = -\frac{qV}{kT} + \ln n_0$$

on choisit potentiel nul pour distribution non perturbée $n = n_0$

$$\Rightarrow \boxed{n = n_0 e^{-\frac{qV}{kT}}}$$

1°)

(a) $\rho(r) = -e n_e(r) + e n_e(r) = e n_0 e^{-\frac{eV}{kT}} - e n_0 e^{\frac{eV}{kT}}$

$$\Rightarrow \boxed{\rho(r) = -2 e n_0 \,\text{sh}\left(\frac{eV}{kT}\right)}$$

<u>Pour $eV \ll kT$</u> : $\boxed{\rho(r) \# -2 \frac{e^2 n_0 V(r)}{kT}}$

(b) <u>Eq. de Poisson</u> : $\Delta V = -\frac{\rho}{\varepsilon_0}$ soit $\frac{1}{r}\frac{d^2(rV)}{dr^2} = -\frac{2 n_0 e^2}{\varepsilon_0 kT} V(r)$.

$$\Rightarrow \frac{d^2(rV(r))}{dr^2} + \frac{2 n_0 e^2}{\varepsilon_0 kT}(rV(r)) = 0 .$$

on pose $\boxed{d_D = \sqrt{\frac{\varepsilon_0 kT}{2 n_0 e^2}}}$ alors : $rV(r) = A e^{-r/d_D} + B e^{r/d_D}$.

$$\Rightarrow \boxed{V(r) = \frac{A}{r} e^{-r/d_D} + \frac{B}{r} e^{r/d_D}}$$

AN : ionosphère : $d_D = 0,81$ mm.

plasma de synthèse : $d_D = 7,4\ \mu m$.

(c) Si $r \to \infty$ : $V \to 0$ donc $B = 0$.

Si $r \to 0$ : ($r \ll d_D$) on retrouve le potentiel coulombien de l'ion

$$V \to \frac{e}{4\pi\varepsilon_0 r} \quad \Rightarrow A = \frac{e}{4\pi\varepsilon_0}$$

D'où $\boxed{V(r) = \frac{e}{4\pi\varepsilon_0 r} e^{-r/d_D}}$

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2°

$$\boxed{\vec{E}(r) = -\left(\frac{\partial V}{\partial r}\right)\vec{u}_r = \frac{e}{4\pi\varepsilon_0 r^2}\left(1+\frac{r}{d_D}\right)e^{-r/d_D}\,\vec{u}_r}$$

Th. de Gauss : $Q(r) = \varepsilon_0 \oint_S \vec{E}.\vec{dS} = \frac{e\,\varepsilon_0}{4\pi\varepsilon_0 r^2} 4\pi r^2 \left(1+\frac{r}{d_D}\right) e^{-r/d_D}$.

$$\Rightarrow \boxed{Q(r) = e\left(1+\frac{r}{d_D}\right)e^{-r/d_D}}$$

$r \to \infty$ : $\frac{r}{d_D} \gg 1$ $Q(r) \to 0$ neutralité électrique du plasma.

$r \to 0$ : $\frac{r}{d_D} \ll 1$ $Q(r) \to e$ ion seul.

$d_D$ représente l'ordre de grandeur de la distance sur laquelle la densité locale de charge est perturbée autour de l'ion.

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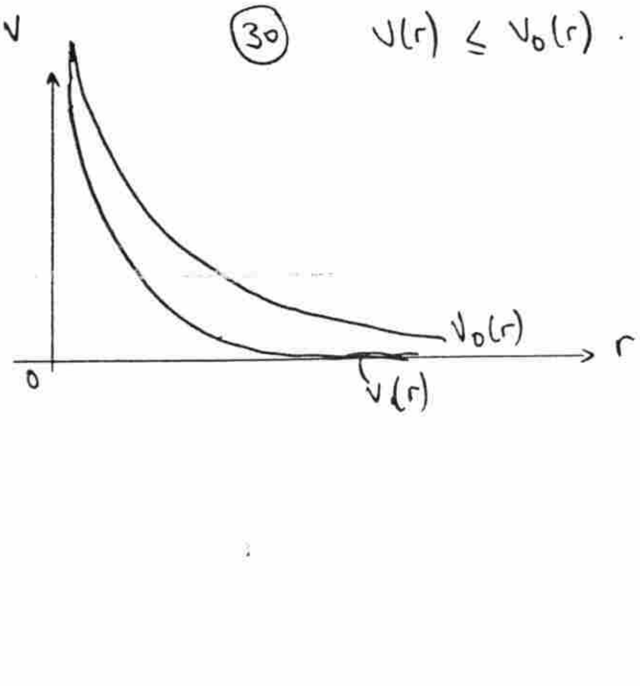
3° $V(r) \leq V_0(r)$.




Loin de l'ion central, $V(r)$ décroît bcp plus vite que $V_0(r)$ : la charge opposée à celle de l'ion central, qui l'entoure, fait écran.

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V vérifie : $\begin{cases} \Delta V = 0 \\ V(0,y) = V(a,y) = 0 \\ V(x,0) = V_0 \end{cases}$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \qquad V = f(x)\,g(y)$$

V ne dépend pas de z par invariance de la distrou selon z.

(*) $f''(x)\,g(y) + f(x)\,g''(y) = 0$.

$$\Rightarrow \quad f''(x) + \underbrace{\frac{g''(y_0)}{g(y_0)}}_{K} f(x) = 0.$$

1er cas : $K < 0$ $\quad f(x) = \alpha e^{Kx} + \beta e^{-Kx}$

ne satisfait pas aux CL.

2e cas : $K = 0$ $\quad g(x) = \alpha x + \beta$ $\quad$ idem

3e cas : $K > 0$. $\quad f(x) = \alpha \cos Kx + \beta \sin Kx$

.....

$V(0,y) = 0 \quad \forall y \Rightarrow f(0) = 0 = \alpha$

$V(a,y) = 0 \quad \forall y \Rightarrow f(a) = 0 = \beta \sin Ka$.

$\Rightarrow \quad Ka = n\pi \quad n \in \mathbb{N}$.

$\Rightarrow \quad K = \frac{n\pi}{a}.\ n \in \mathbb{N}^*$.

D'où $\underline{f(x) = f_0 \sin \frac{n\pi}{a} x \ (n \in \mathbb{N}^*)}$.

(*) $f''(x)\,g(y) + f(x)\,g''(y) = -\frac{n^2\pi^2}{a^2} g(y) + g''(y) = 0$.

$$\Rightarrow \quad g(y) = \lambda e^{-\frac{n\pi y}{a}} + \mu e^{+\frac{n\pi y}{a}}.$$

$\left.\begin{matrix} y \to \infty \\ V \to 0 \end{matrix}\right\} \Rightarrow g(y) \underset{y \to \infty}{\longrightarrow} 0 \quad$ d'où $\mu = 0$.

→ solutions pour n donné :

$$V_n(x,y) = f_n \sin\frac{n\pi x}{a}\, e^{-\frac{n\pi y}{a}}.$$

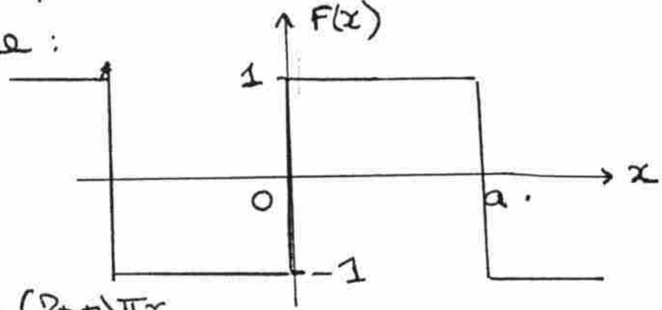
ces solution ne satisfont pas à la condition $V(x,0) = V_0 \ \forall x$.

② on envisage la superposition des $V_n$ :

$$V(x,y) = \sum_{n=1}^{\infty} f_n \sin\left(\frac{n\pi x}{a}\right) e^{-\frac{n\pi y}{a}}$$

$$V(x,0) = \sum_{n=1}^{\infty} f_n \sin\left(\frac{n\pi x}{a}\right) = V_0 \text{ pour } 0<x<a.$$

si on écrit la décomposition en série de Fourier de la fonction périodique :




$$F(x) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{2p+1} \sin\frac{(2p+1)\pi x}{a}$$

on identifie alors $f_n$ à $\frac{4}{\pi} V_0 \frac{1}{2p+1}$ pour $n = 2p+1$.

D'où $$\boxed{V(x,y) = \frac{4V_0}{\pi}\sum_{n=1}^{\infty} \frac{1}{2p+1} \sin\frac{(2p+1)\pi x}{a}\, e^{-\frac{(2p+1)\pi y}{a}}.}$$