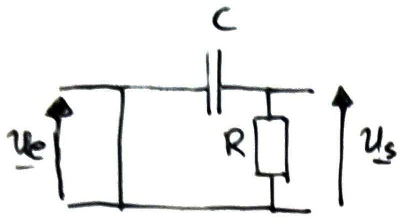
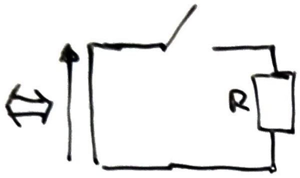
5 Filtre passe-haut

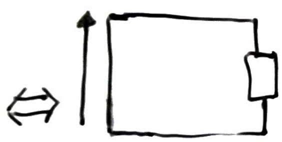


- $\omega \to 0 \Rightarrow |\underline{Z}_c| \to \infty \Rightarrow$ (⇔) **BF**



$\underline{u}_s \to 0$

- $\omega \to \infty \Rightarrow |\underline{Z}_c| \to 0 \Rightarrow$ (⇔) **HF**



$\underline{u}_s = \underline{u}_e$

Le filtre laisse bien passer les hautes fréquences et atténue les basses fréquence

$$\underbrace{\hphantom{Le filtre laisse bien passer}}_{\text{Filtre passe-haut}}$$

Erratum

" 4. Filtre " $\rightsquigarrow$ 4. Filtre passe-bas

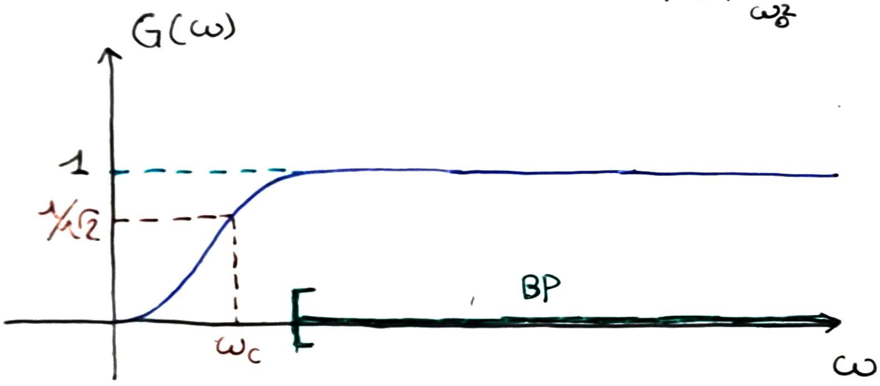
- Étude

$$\underline{H} = \frac{\underline{u_s}}{\underline{u_e}} = \frac{R}{R + \frac{1}{jC\omega}} = \frac{jRC\omega}{1 + jRC\omega}$$

$$\omega_0 := \frac{1}{RC}$$

$$\underline{H} = \frac{j\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_0}}$$

Graphe de $G(\omega) = |\underline{H}| = \dfrac{\frac{\omega}{\omega_0}}{\sqrt{1 + \frac{\omega^2}{\omega_0^2}}}$



- Diagramme de Bode

$$\underline{H} = \frac{j\frac{\omega}{\omega_0}}{1 + j\frac{\omega}{\omega}}$$

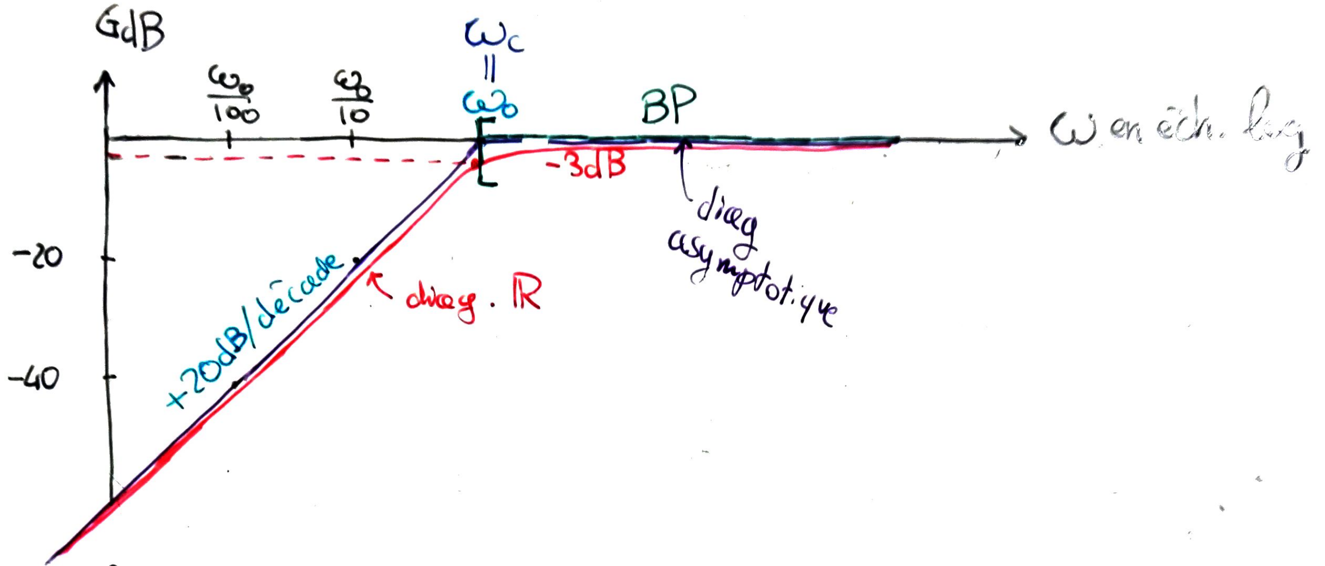
à $\omega \ll \omega_0$: $\underline{H} \sim j\frac{\omega}{\omega_0} \Rightarrow$

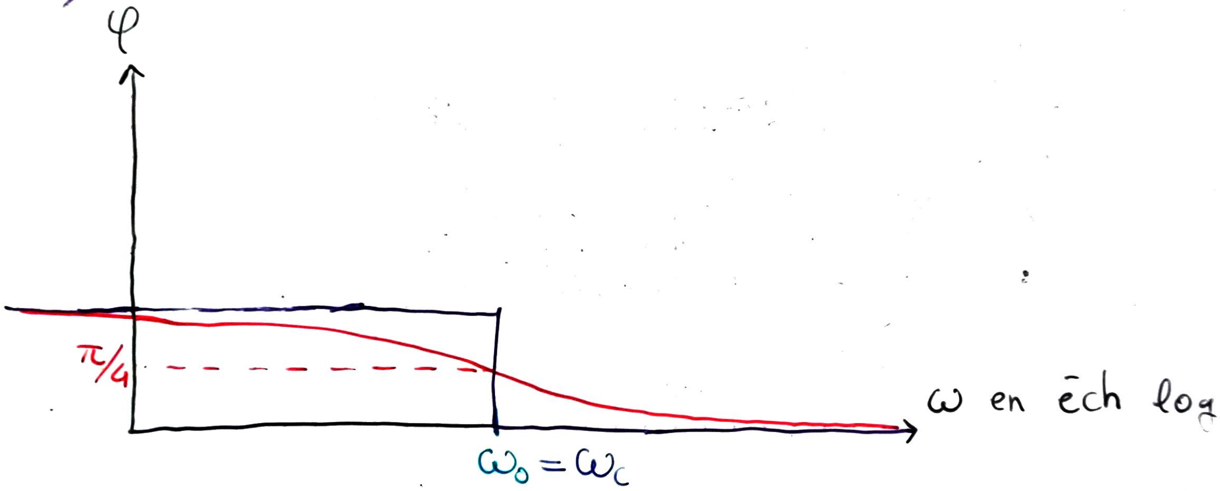
On a $\begin{cases} G_{dB} = 20 \log |\underline{H}| = 20 \log \frac{\omega}{\omega_0} \\ \arg \underline{H} = \frac{\pi}{2} \end{cases}$

à $\omega \gg \omega_0$: $\underline{H} \sim 1 \Rightarrow \begin{cases} G_{dB} = 20 \log 1 = 0 \\ \varphi = 0 \end{cases}$

$\underline{\text{à } \omega = \omega_0}$ $\quad \underline{H} = \dfrac{j}{1+j} \Rightarrow \begin{cases} G_{dB} = 20 \log = -10 \log 2 \\ \varphi = \end{cases}$

$$\begin{cases} G_{dB} = 20 \log \dfrac{1}{\sqrt{2}} = -10 \log 2 = -3 dB \\ \varphi = \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{4} \end{cases}$$





- pulsation de coupure à $-3dB$: $\boxed{\omega_c}$

$\boxed{\omega_c} / \quad G(\omega_c) = \dfrac{G_{max}}{\sqrt{2}} \Leftrightarrow G_{dB}(\omega_c) = G_{dB_{max}} + 20 \log \dfrac{1}{\sqrt{2}}$

$= G_{dB_{max}} - 3 \, dB$

- On lit $\omega_0 = \omega_c$

- Vérifie par le calcul

$$G(\omega_c) = \frac{\frac{\omega_c}{\omega_0}}{\sqrt{1+\left(\frac{\omega_c}{\omega_0}\right)^2}} = \frac{G_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow 2\left(\frac{\omega_c}{\omega_0}\right)^2 = 1+\left(\frac{\omega_c}{\omega_0}\right)^2$$

$$\Rightarrow \left(\frac{\omega_c}{\omega_0}\right)^2 = 1$$

$$\Rightarrow \omega_c = \omega_0$$

- Bande passante à $-3dB$

$$G \geqslant \frac{G_{max}}{\sqrt{2}} \Leftrightarrow G_{dB} \geqslant G_{dB_{max}} - 3dB$$

rem q

$$\omega \gg \omega_0 \Rightarrow \underline{H} = j\frac{\omega}{\omega_0} = \frac{\underline{u_s}}{\underbrace{\underline{u_e}}} = \frac{\underline{U_s}}{\underline{U_e}}$$

$$\underline{u_s} = j\frac{\omega}{\omega_0}\underline{u_e} = \frac{1}{\omega_0}\frac{d\underline{u_e}}{dt}$$

$Re(\cdot)$

$$u_s(t) = \frac{1}{\omega_0}\dot{u_e}$$

En BF, le filtre CR est DÉRIVATEUR

6 Exercice

$$\underline{H} = \frac{\underline{u_s}}{\underline{u_e}} = \frac{r+\frac{1}{jC\omega}}{R+r+\frac{1}{jC\omega}} = \frac{1+jrC\omega}{1+j(r+R)\omega} = \frac{1+j\frac{\omega}{\omega_0}}{1+j\frac{\omega}{\omega_1}} \quad \text{où} \begin{cases}\omega_0 = \frac{1}{rC}\\ \omega_1 = \frac{1}{(r+R)C}\end{cases}$$

$$\Rightarrow \omega_1 < \omega_0$$

On décompose $\underline{H}$

$$\underline{H} = \underline{H_0}\ \underline{H_1}$$

- $G_{dB} = 20 \log |\underline{H}| = 20 \log |\underline{H_0}| + 20 \log |\underline{H_1}|$

$$= G_{0dB} + G_{1dB}$$

- $\varphi = \arg \underline{H} = \arg \underline{H_0} + \arg \underline{H_1}$

On tracera les diagrammes de Bode asymptotiques pour $\underline{H_0}$ et $\underline{H_1}$

et on les sommera graphiquement

où $\begin{cases} \underline{H_0} = 1 + j\frac{\omega}{\omega_0} \\ \underline{H_1} = \frac{1}{1+j\frac{\omega}{\omega_1}} \end{cases}$

- $\boxed{\underline{H_0}} = 1 + j\frac{\omega}{\omega_0}$

$\omega \ll \omega_0 \qquad \underline{H_0} \sim 1 \Rightarrow \begin{cases} G_{0dB} = 20 \log |\underline{H_0}| = 0 \\ \arg \underline{H_0} = 0 \end{cases}$

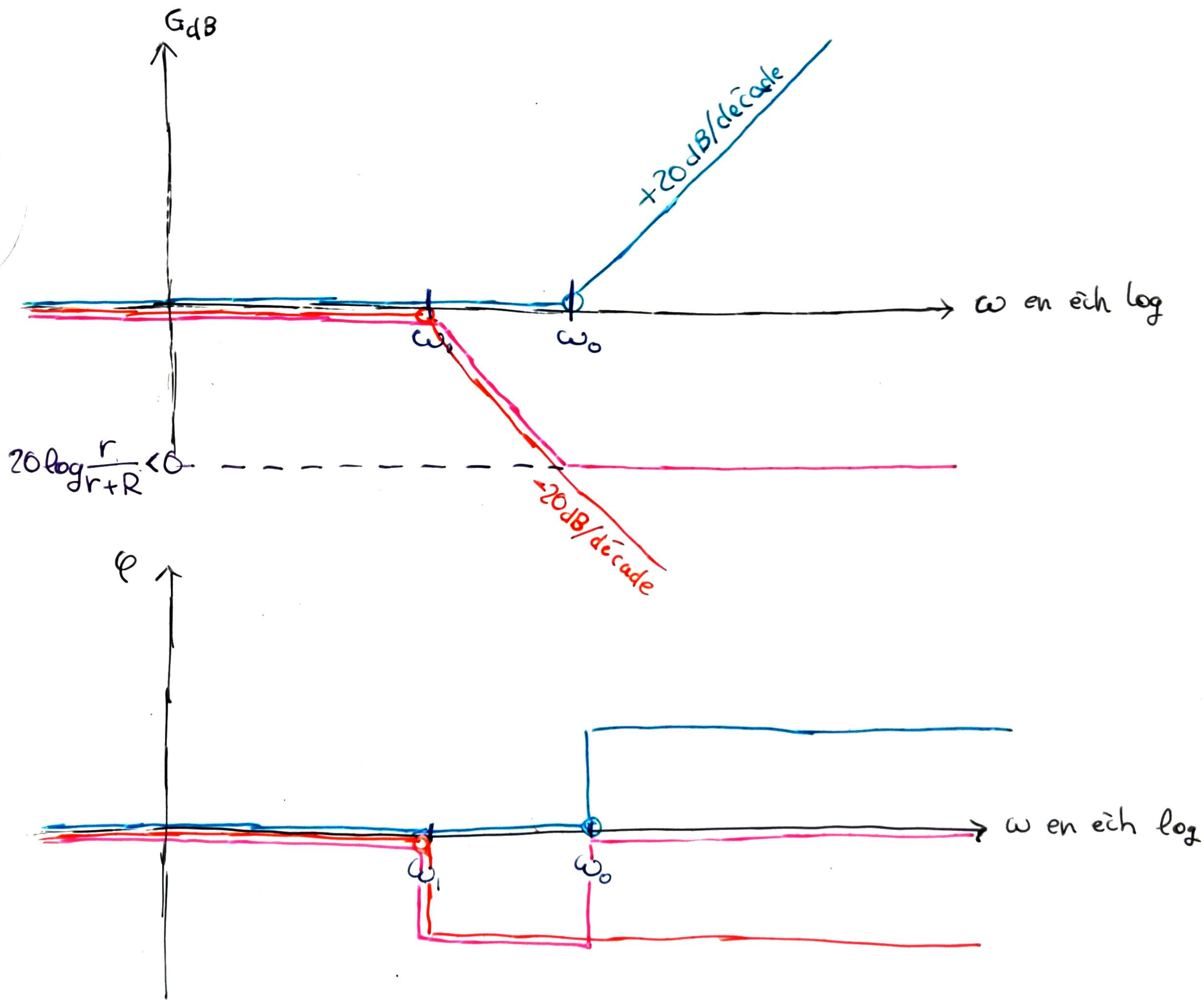
$\omega \gg \omega_0 \qquad \underline{H_0} \sim j\frac{\omega}{\omega_0} \Rightarrow \begin{cases} G_{0dB} = 20 \log \frac{\omega}{\omega_0} \\ \arg \underline{H_0} = \frac{\pi}{2} \end{cases}$

- $\boxed{\underline{H_1}} = \frac{1}{1+j\frac{\omega}{\omega_1}}$

$\omega \ll \omega_1 \qquad \underline{H_1} \sim 1 \Rightarrow \begin{cases} G_{dB} = 0 \\ \arg \underline{H_1} = 0 \end{cases}$

$\omega \gg \omega_1 \qquad \underline{H_1} \sim \dfrac{1}{j\frac{\omega}{\omega_1}} \Rightarrow \begin{cases} G_{dB} = -20 \log \frac{\omega}{\omega_1} \\ \arg \underline{H_1} = -\frac{\pi}{2} \end{cases}$

Diagramme de Bode



Il laisse passer les BF, atténue les HF d'un facteur constant :

$$\underline{H} \underset{+\infty}{\sim} \frac{jrC\omega}{j(R+r)C\omega} = \boxed{\frac{r}{r+R}} < 1$$

BP : $\omega / U \geq \frac{U_{max}}{\sqrt{2}}$

$$\frac{\cancel{I_0}}{\sqrt{\frac{1}{R^2}+\left(C\omega-\frac{1}{L\omega}\right)^2}} = \frac{R\cancel{I_0}}{\sqrt{2}}$$

$$2 = R^2\left(\frac{1}{R^2}+\left(C\omega-\frac{1}{L\omega}\right)^2\right)$$

$$\frac{1}{R^2} = \left(C\omega-\frac{1}{L\omega}\right)^2$$

$$C\omega-\frac{1}{L\omega} = \pm\frac{1}{R}$$

RLω

$$RLC\omega^2 = \pm L\omega - R = 0$$

$$\Delta = L^2 + 4R^2LC > 0$$

sol. $\omega = \frac{\pm L \pm\sqrt{\Delta}}{2RLC}$

$\omega_{c_2} = \frac{+L+\sqrt{\Delta}}{2RLC}$ $\omega_{c_1} = \frac{-L+\sqrt{\Delta}}{2RLC}$ $(\sqrt{\Delta} > L)$

$$\Delta\omega = \omega_{c_2} - \omega_{c_1} = \frac{\cancel{L}+\cancel{\sqrt{\Delta}}+\cancel{L}-\cancel{\sqrt{\Delta}}}{\cancel{2}R\cancel{L}C} = \frac{1}{RC}$$

$$Q = A_c = \frac{\omega_0}{\Delta\omega} = \frac{RC}{\sqrt{LC}} = R\sqrt{\frac{C}{L}}$$

R grand quand $\Delta\omega$ petit : filtre sélectif