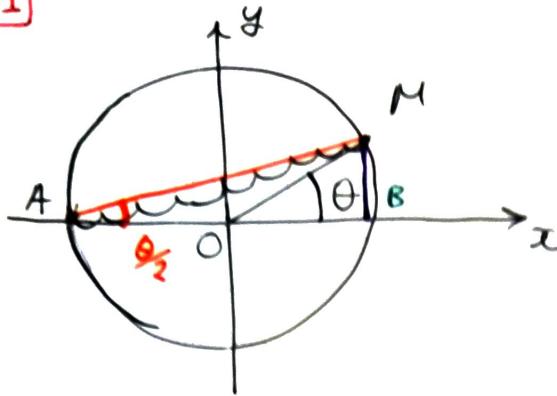


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$$\begin{aligned} E_p(M) &= E_{p_p} + E_{p_e} \\ &= mgy + \frac{1}{2}k(AM - l_0)^2 + \text{const} \end{aligned}$$

avec  $y = a \cdot \sin \theta$

$$AM = 2a \cdot \cos \frac{\theta}{2}$$

$$E_p = mg a \sin \theta + \frac{1}{2}k(2a \cos \frac{\theta}{2} - l_0)^2 + \text{const}$$

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$$\bar{E}_p(\theta) = mg a \sin \theta + \frac{1}{2}k a^2 \cos^2 \frac{\theta}{2}$$

$$\text{En position d'éq. } \Leftrightarrow \left( \frac{d\bar{E}_p}{d\theta} \right)_{\theta_e} = 0$$

$$\begin{aligned} \frac{d\bar{E}_p}{d\theta} &= mg a \cos \theta + \frac{1}{2}k a^2 2 \cos \frac{\theta}{2} \cdot \left( -\frac{1}{2} \sin \frac{\theta}{2} \right) \\ &= mg a \cos \theta - \frac{1}{2}ka^2 \sin \theta \end{aligned}$$

$$\left( \frac{d\bar{E}_p}{d\theta} \right)_{\theta_e} = 0 \Leftrightarrow mg a \cos \theta_e - \frac{1}{2}ka^2 \sin \theta_e = 0$$

$$\Leftrightarrow \tan \theta_e = \frac{mg}{ka}$$

$$\Leftrightarrow \begin{cases} \theta_e = \arctan \frac{mg}{ka} \\ \text{ou} \end{cases} \quad (*)$$

$$\theta_e = \arctan \frac{mg}{ka} + \pi \quad (**)$$

$$\begin{aligned} \left( \frac{d^2\bar{E}_p}{d\theta^2} \right)_{\theta_e} &= -mg \cdot a \cdot \sin \theta_e - \frac{1}{2}ka^2 \cos \theta_e \\ &= -\frac{1}{2}ka^2 \cos \theta_e \left( 1 + \frac{mg}{ka} \tan \theta_e \right) \end{aligned}$$

$$= -\frac{1}{2}ka^2 \cos \theta_e \left( 1 + \frac{mg}{ka} \frac{mg}{ka} \right)$$

$\Rightarrow (*) \Rightarrow < 0$  et  $(**) \Rightarrow > 0$

unstable

stable