LE BIHAN
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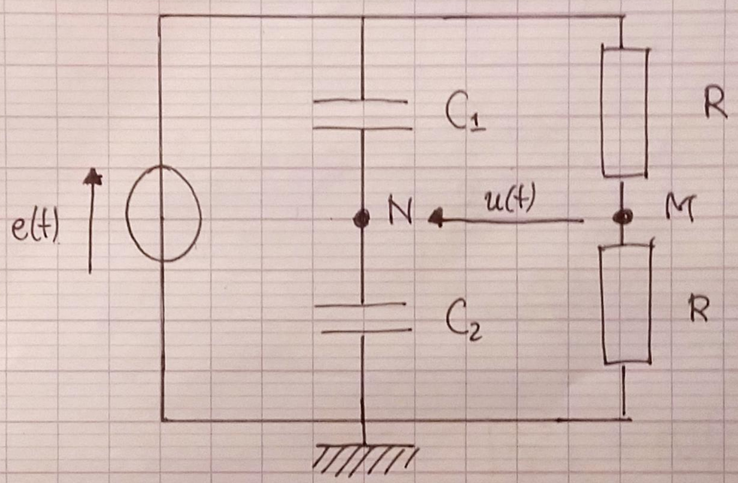
DM Vacances — Électricité

1. $\underline{u}(t) = U e^{j(\omega t + \varphi)} = \underline{U} e^{j\omega t}$

$\underline{U} = U e^{j\varphi}$

2.



Par diviseur de courant :

$$V_N - 0 = \frac{\frac{1}{jC_2\omega}}{\frac{1}{jC_1\omega} + \frac{1}{jC_2\omega}} \underline{e}(t) = \frac{1}{\frac{C_2}{C_1} + 1} \underline{e}(t)$$

$$V_M - 0 = \frac{R}{R+R} \underline{e}(t) = \frac{1}{2} \underline{e}(t)$$

Enfin :

$$\underline{u}(t) = V_N - V_M = \left(\frac{1}{\frac{C_2}{C_1} + 1} - \frac{1}{2} \right) \underline{e}(t)$$

$$\text{ie} \quad \boxed{\underline{U} \cancel{e^{j\omega t}} = \left(\frac{C_1}{C_2 + C_1} - \frac{1}{2} \right) E \cancel{e^{j\omega t}}}$$

3.

$$\underline{U} = \left(\frac{\cancel{R_0}\left(1+\frac{2x}{L}\right)}{\cancel{R_0}\left(1-\frac{2x}{L}\right)+\cancel{R_0}\left(1+\frac{2x}{L}\right)} - \frac{1}{2} \right) E$$

$$= \left(\frac{1+\frac{2x}{L}}{1-\cancel{\frac{2x}{L}}+1+\cancel{\frac{2x}{L}}} - \frac{1}{2} \right) E$$

$$= \frac{1+\frac{2x}{L}-1}{2} E$$

$$= \frac{Ex}{L}$$

donc $|\underline{U}| = \left|\frac{Ex}{L}\right|$

ie $U = \frac{Ex}{L}$

4.

$$\sigma_U = \left|\frac{dU}{dx}\right| = \left|\frac{E}{L}\right|$$

Ce résultat nous montre qu'il y a, au signe près, une relation de proportionalité entre la sensibilité de la sortie et l'amplitude d'entrée.
Il nous permettra d'interpréter le signal de sortie.

5. On a

$$\varphi(x) = \arg \underline{U}$$

$$= \arg \frac{Ex}{L}$$

$$\equiv \begin{cases} 0 & \text{si } x \in \mathbb{R}_+ \\ \pi & \text{sinon} \end{cases} \quad [2\pi]$$

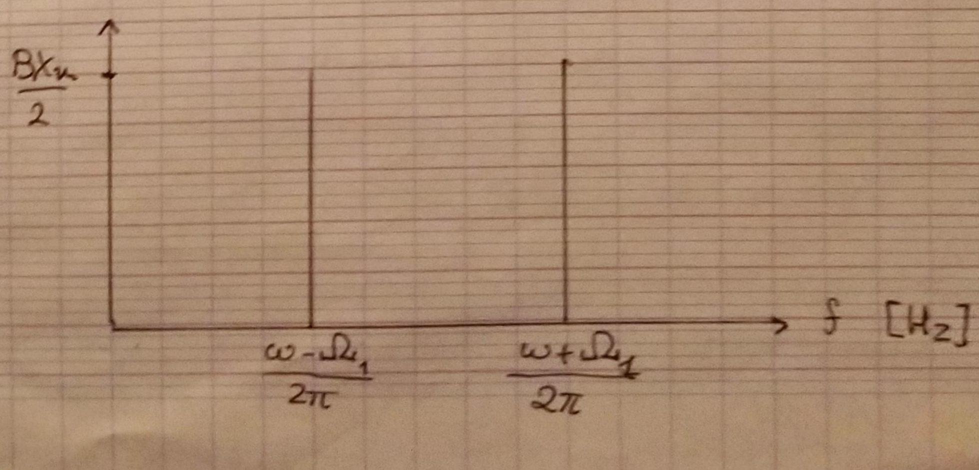
Le déphasage nous donne le sens de déplacement : $\begin{cases} \varphi(x) \equiv 0\ [2\pi] \Leftrightarrow \text{selon } \vec{z} \\ \varphi(x) \equiv \pi\ [2\pi] \Leftrightarrow \text{selon } -\vec{z} \end{cases}$

6. a

$$u(t) = B\cos \omega t\, X_m \cos \Omega_1 t$$

$$= B X_m \cos(\omega t) \cos(\Omega_1 t)$$

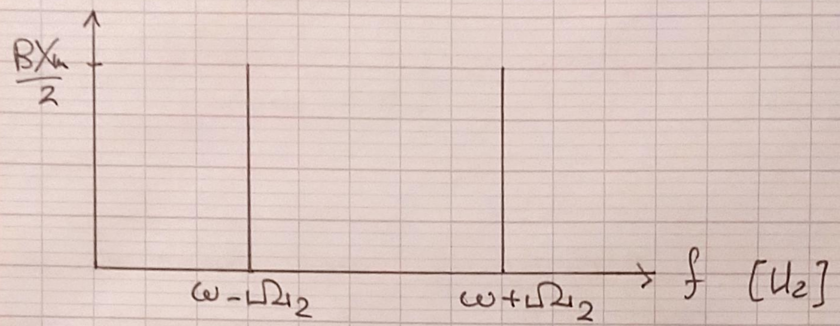
$$= \frac{B X_m}{2}\left[\cos((\omega + \Omega_1)t) + \cos((\omega - \Omega_1)t)\right]$$



b

$$u(t) = -BX_m \cos(\omega t)\cos(\Omega_2 t)$$

$$= -\frac{BX_m}{2}\left[\cos((\omega+\Omega_2)t) + \cos((\omega-\Omega_1)t)\right]$$



7. On compte 20 oscillations électriques pour une mécanique sur a).

On compte 2 oscillations mécaniques pour le signal a).

On a donc, sur a):

$$\boxed{\begin{aligned} f_e &= \frac{2\cdot 20}{0,1} = 400\ \text{Hz} \\ f_m &= \frac{2}{0,1} = 20\ \text{Hz} \end{aligned}}$$

Sur b), on compte 5,2 oscillations mécaniques, d'où

$$\boxed{f_m = \frac{5,2}{0,1} = 52\ \text{Hz}}$$

8. On a

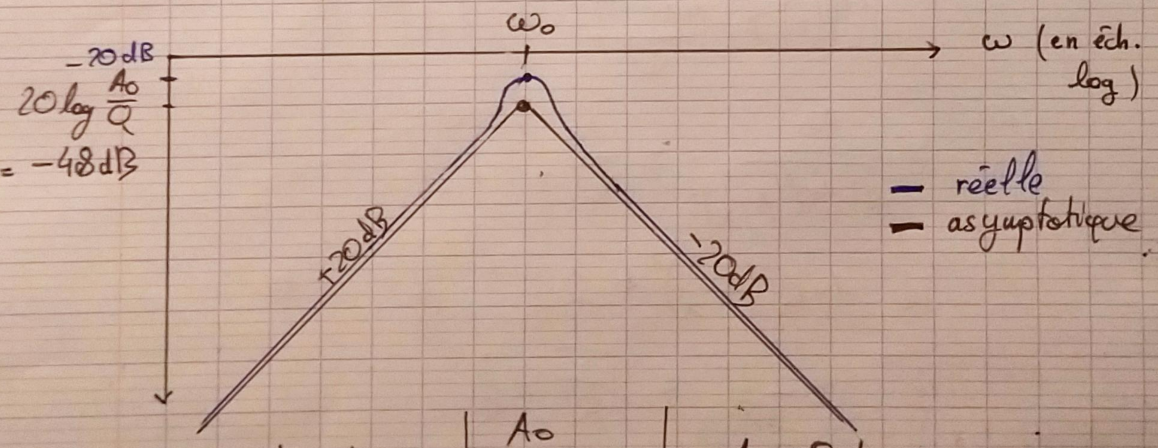
$$\begin{cases} \Omega_1 = 2\pi f_{m_a} = 126 \text{ rad}\cdot s^{-1} \\ \Omega_2 = 2\pi f_{m_b} = 327 \text{ rad}\cdot s^{-1} \end{cases}$$

9. a.

$$\begin{cases} \underline{H} \xrightarrow[\omega \to 0]{} \dfrac{A_0}{-j\frac{Q}{x}} = j\dfrac{A_0}{Q}x \\ \underline{H} \xrightarrow[\omega \to \infty]{} \dfrac{A_0}{jQx} = -j\dfrac{A_0}{Qx} \end{cases}$$

b.

$$\begin{cases} \text{BF} & G_{dB} = 20\log\left(\dfrac{A_0}{Q}x\right) = 20\log\dfrac{A_0}{Q} + 20\log x \\ \text{HF} & G_{dB} = 20\log\left(\dfrac{A_0}{Qx}\right) = 20\log\dfrac{A_0}{Q} - 20\log x \end{cases}$$



$$|\underline{H}(\omega_0)| = \left|\frac{A_0}{1+jQ(1-\frac{1}{1})}\right| = A_0 = 0{,}1$$

$$\Rightarrow G_{dB}(\omega_0) = 20\log 0{,}1 = -20\text{ dB}$$

c. c'est un passe-bande.

le maximum est atteint en $x = 1$, il y a donc résonance en $x = 1$.

$$\arg u_2 - \arg u_1 = \arg \frac{u_2}{u_1}$$

$$= \arg \underline{H}$$

$$= \arg A_0 - \arg\left(1 + \cancel{jQ\left(1 - \frac{1}{1}\right)}\right)$$

$$= 0 - 0 \quad \text{car } A_0 > 0$$

$$= 0$$

d.

$$|\underline{H}|(x) \geq \frac{\max |\underline{H}|}{\sqrt{2}}$$

ie $$\frac{A_0}{\sqrt{1 + Q^2\left(x - \frac{1}{x}\right)^2}} \geq \frac{A_0}{\sqrt{2}}$$

ie $$1 + Q^2\left(x - \frac{1}{x}\right)^2 \leq 2$$

ie $$x - \frac{1}{x} \leq \sqrt{\frac{1}{Q^2}} = \frac{1}{Q}$$

ie $$x^2 - 1 - \frac{x}{Q} \leq 0$$

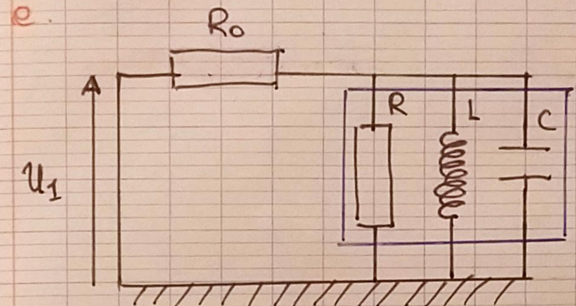
$$\Delta = \frac{1}{Q^2} + 4 \; ; \quad x_{1,2} = \frac{1}{2Q} \pm \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4}$$

$$\Delta x = x_1 - x_2 = \frac{1}{2Q} + \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4} - \frac{1}{2Q} + \frac{1}{2}\sqrt{\frac{1}{Q^2} + 4}$$

d'où

$$\boxed{\Delta x = \sqrt{\frac{1}{Q^2} + 4}}$$

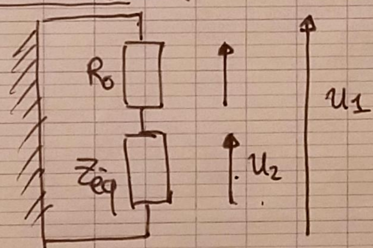
e.



$$Z_{éq} = \frac{1}{\frac{1}{R} + \frac{1}{jL\omega} + jC\omega} = \frac{1}{\frac{1}{R} + j\left(C\omega - \frac{1}{L\omega}\right)}$$

u_2

Diviseur de tension :



$$u_2 = \frac{Z_{éq}}{R_0 + Z_{éq}} u_1$$

ie $$\underline{H} = \frac{1}{\frac{R_0}{Z_{éq}} + 1}$$

$$\frac{R_0}{Z_{éq}} = \frac{R_0}{R} + jR_0\left(C\omega - \frac{1}{L\omega}\right) = \frac{R_0}{R} + jR_0\sqrt{\frac{C}{L}}\left(C\sqrt{\frac{L}{C}}\omega - \frac{1}{L}\sqrt{\frac{L}{C}}\frac{1}{\omega}\right)$$

Posons $\omega_0 := \frac{1}{\sqrt{LC}}$

$$= \frac{R_0}{R} + jR_0\sqrt{\frac{C}{L}}\left(\sqrt{LC}\,\omega - \sqrt{\frac{1}{LC}}\frac{1}{\omega}\right)$$

$$= \frac{R_0}{R} + jR_0\sqrt{\frac{C}{L}}\left(\underbrace{\frac{\omega}{\omega_0}}_{x} - \underbrace{\frac{\omega_0}{\omega}}_{\frac{1}{x}}\right)$$

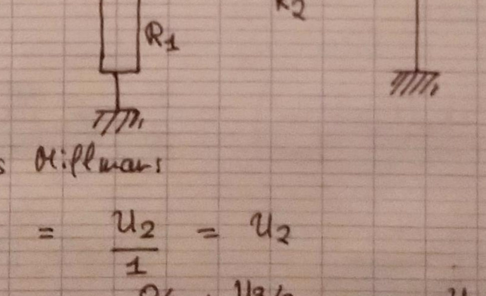
$$\underline{H} = \frac{1}{1+\frac{R_0}{R}+jR_0\sqrt{\frac{C}{L}}\left(x-\frac{1}{x}\right)}$$

$$= \frac{1+R_0/R}{1+j\frac{R_0\sqrt{\frac{C}{L}}}{1+\frac{R_0}{R}}\left(x-\frac{1}{x}\right)}$$

$$=: \frac{A_0}{1+jQ\left(x-\frac{1}{x}\right)}$$

$$\boxed{\begin{cases} A_0 = 1+R_0/R \\ \omega_0 = \sqrt{\frac{1}{LC}} \\ Q = \frac{RR_0\sqrt{\frac{C}{L}}}{R+R_0} = \sqrt{\frac{C}{L}}\frac{RR_0}{R+R_0} \end{cases}}$$

10. a.



D'après Millman :

$$\begin{cases} V_+ = \frac{u_2}{1} = u_2 \\ V_- = \frac{0/R_1 + u_3/R_2}{1/R_1 + 1/R_2} = \frac{u_3}{R_2/R_1 + 1} = \frac{R_1}{R_2+R_1}u_3 \end{cases}$$

$$V_+ = V_- \quad \text{donc} \quad u_2 = \frac{R_1}{R_2+R_1} u_3$$

$$\text{ie} \quad \underline{G} = \frac{R_1}{R_2+R_1}$$

b.

$$K = |\underline{G}| = \left| \frac{R_1}{R_2+R_1} \right| = \frac{R_1}{R_2+R_1}$$

11.a. On sait que $\underline{u_1} = \underline{u_3}$

$$\begin{cases} \underline{u_3} = K \underline{u_2} \\ \underline{u_2} = \underline{H}\, \underline{u_1} = \underline{H}\, \underline{u_3} \end{cases}$$

donc $\underline{u_3} = K\underline{H}\, \underline{u_3}$.

$$\underline{u_3} = K\underline{H}\, \underline{u_3} = \frac{KA_0}{1+jQ(x-\frac{1}{x})} \underline{u_3}$$

$$\text{ie} \quad \left(1+jQ(x-\tfrac{1}{x})\right) \underline{u_3} = KA_0 \underline{u_3}$$

$$\text{ie} \quad (1-KA_0)\underline{u_3} + \frac{Q}{\omega_0} \dot{\underline{u_3}} + Q\omega_0 \overset{|}{\underline{u_3}}{}^{*} = 0$$

$$\text{ie} \quad (1-KA_0)\dot{\underline{u_3}} + \frac{Q}{\omega_0} \ddot{\underline{u_3}} + Q\omega_0 \underline{u_3} = 0$$

$$\text{ie} \quad \boxed{\underline{u_3} + \frac{1-KA_0}{Q\omega_0} \dot{\underline{u_3}} + \frac{1}{\omega_0^2} \ddot{\underline{u_3}} = 0}$$

b.

On cherche à ce que u_3 soit en régime pseudo-périodique, i.e. que le discriminant de l'équation caractéristique soit strictement négatif

* notation de Newton pour les primitives : $\overset{|}{u}$

Équation caractéristique :

$$X^2 + \frac{1-KA_0}{Q\omega_0}X + \frac{1}{\omega_0^2} = 0$$

Condition :

$$\Delta = \frac{(1-KA_0)^2}{Q^2\omega_0^2} - \frac{4}{\omega_0^2} < 0$$

$$\text{ie} \quad \frac{(1-KA_0)^2}{Q^2\cancel{\omega_0^2}} < \frac{4}{\cancel{\omega_0^2}}$$

$$\text{ie} \quad (1-KA_0)^2 < 4Q^2$$

$$\text{ie} \quad -KA_0 < 4Q^2 - 1$$

$$\text{ie} \quad \boxed{KA_0 > 1 - 4Q^2}$$

Solutions (en supposant $\Delta < 0$) :

$$X_{1,2} = -\frac{1-KA_0}{2Q\omega_0} \pm j\frac{1}{2}\sqrt{\frac{4Q^2-(1-KA_0)^2}{Q^2\omega_0^2}}$$

$$\text{donc} \quad \underline{u}_3(t) = \left(A\cos\Big(\underbrace{\frac{1}{2}\frac{\sqrt{4Q^2-(1-KA_0)^2}}{Q\omega_0}}_{\omega}t\Big) + B\sin\Big(\underbrace{\frac{1}{2}\frac{\sqrt{4Q^2-(1-KA_0)^2}}{Q\omega_0}}_{\omega}t\Big)\right) \cdot e^{\frac{KA_0-1}{2Q\omega_0}}$$

$$\text{d'où} \quad \boxed{f_0 = \frac{1}{4\pi Q\omega_0}\sqrt{4Q^2-(1-KA_0)^2}}$$

12. a. On a $x \ll l$

ie $\frac{x}{l} \ll 1$.

$$f_{osc} = \frac{D}{\sqrt{C}} = \frac{D}{\sqrt{C_0\left(1-\frac{x}{l}\right)}} = \frac{D}{\sqrt{C_0}\sqrt{1-\frac{x}{l}}}$$

$$= \frac{D}{\sqrt{C_0}}\left(1-\frac{x}{l}\right)^{-\frac{1}{2}}$$

$$\approx \frac{D}{\sqrt{C_0}}\left(1-\left(-\frac{1}{2}\right)\frac{x}{l}\right)$$

$$= \frac{D}{2\sqrt{C_0}\,l}x + \frac{D}{\sqrt{C_0}}$$

On a $\boxed{\begin{cases} \frac{D}{2\sqrt{C_0}\,l} =: a \\ \frac{D}{\sqrt{C_0}} =: b \end{cases}}$

b.

$$\Delta f = f_{osc} - f_{or}$$

$$= f_{osc} - f_{osc}(0)$$

$$= \frac{D}{2\sqrt{C_0}\,l}x + \cancel{\frac{D}{\sqrt{C_0}}} - \cancel{\frac{D}{\sqrt{C_0}}}$$

$\Delta f(x_{min}) = \Delta_{min}$

ie $\frac{D}{2\sqrt{C_0}\,l}x_{min} = 3\text{ Hz}$ ie $\boxed{x_{min} = \frac{6\sqrt{C_0}\,l}{D} = 1{,}89 \cdot 10^{-4}\text{ m}}$