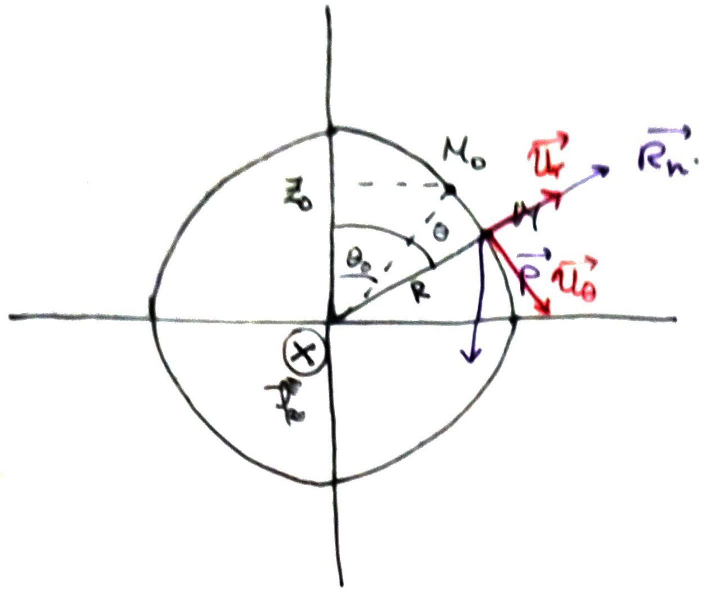
EX P CIN POINT

$\boxed{1}$



$\boxed{1/1}$ $S : \{M\}$; R : Labo, Galiléen; $F : \vec{P}, \vec{R_n}$, $\cancel{\vec{R_t}}$ ← pas de frottements

D'après le TEM :

$$\underset{M_0 \to M}{\Delta E_m} = W(\vec{F}_{dissipative})$$

$$= W(\vec{R_n})$$

$$= 0 \qquad \text{car } \forall t,\ \vec{R_n} \perp \text{déplacement}$$

$$\underset{M_0 \to M}{\Delta E_m} = 0 \Leftrightarrow E_m = \text{const}$$

$\boxed{1/2}$

$$\forall t, \quad E_m = E_c + E_p = \frac{1}{2} m v^2 + mg(z_M - z_{M_0}) + \text{const} \qquad \text{car} \uparrow z$$

$$= \frac{1}{2} m (R\dot{\theta}\vec{u_\theta})^2 + mgR\cos\theta + \text{const}$$

$$E_m = \text{const} = \frac{1}{2} mR^2\dot{\theta}^2 + mgR\cos\theta$$

$$@t=0 \qquad = 0 + mgR\cos\theta$$

$$\frac{1}{2} m R^2 \dot{\theta}^2 + mg R\cos\theta = mg R\cos\theta_0$$

$$\cancel{\frac{1}{2}} \cancel{m} R^{\cancel{2}} \cancel{2} \ddot{\theta} \cancel{\dot{\theta}} - \cancel{m} g \cancel{R} \cancel{\dot{\theta}} \sin\theta = \cancel{- mg R \dot{\theta}_0 \sin\theta_0} \quad (*)$$

$$\Leftrightarrow R\ddot{\theta} - g\sin\theta = 0$$

$$\Leftrightarrow \ddot{\theta} - \frac{g}{R}\sin\theta = 0$$

1/3

d'après la 2e loi de Newton :

$$m\vec{a} = \vec{P} + \vec{R_n}$$

Meth 1 :

$$\Leftrightarrow m\left(l\ddot{\theta}\vec{u_\theta} - l\dot{\theta}^2\vec{u_r}\right) = -mg\cos\theta\,\vec{u_r} + mg\sin\theta\,\vec{u_\theta}$$

On projette

$$\begin{cases} /\vec{u_r} & -ml\dot{\theta}^2 = R_n - mg\cos\theta \\ /\vec{u_\theta} & ml\ddot{\theta} = mg\sin\theta \end{cases}$$

Meth 2

$$\vec{L_0} = \vec{OM} \wedge \underline{\underline{m}}\vec{v} = l\vec{u_r} \wedge (-mg\cos\theta\,\vec{u_r} + mg\sin\theta\,\vec{u_\theta}) = mgl^2\dot{\theta}\vec{k}$$

$$\dot{\vec{L_0}} = ml^2\ddot{\theta}\vec{k}$$

$$\vec{M_0}(\vec{P}) = \vec{OM} \wedge \vec{P} = l\vec{u_r} \wedge (\dots) = mgl\sin\theta\,\vec{k}$$

$$\vec{M_0}(\vec{R_n}) = \vec{0}$$

$$R_n = mg\cos\theta - ml\dot{\theta}^2$$

Pour trouver $\dot{\theta}^2$:

$$\Leftrightarrow \dot{\theta}\, mR\ddot{\theta} = mg\sin\theta\,\dot{\theta}$$

$$\Leftrightarrow \frac{1}{2} mR\dot{\theta}^2 = -mg\cos\theta + C$$

$$\Leftrightarrow \quad 0 = -mg\cos\theta_0 + C \qquad (\dot{\theta}_0 = 0)$$

$$\Leftrightarrow \quad C = mg\cos\theta_0$$

d'où $\quad \frac{1}{2} mR\dot{\theta}^2 = -mg\cos\theta + mg\cos\theta_0 \quad (*)$

Enfin:

$$R_n = mg\cos\theta - mR\dot{\theta}^2$$

$$= mg\cos\theta - 2(-mg\cos\theta + mg\cos\theta_0)$$

$$\Rightarrow R_n = mg(3\cos\theta - 2\cos\theta_0)$$

$\boxed{1/4}$

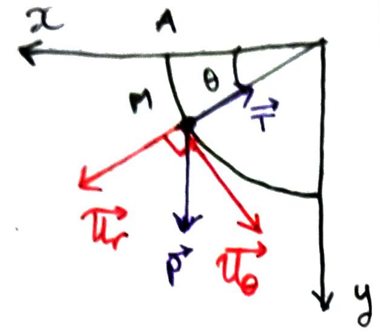
M quitte la sphère lorsqu'il n'y a plus de contact, donc lorsque $\|\vec{R_n}\| = 0$

$$mg(3\cos\theta - 2\cos\theta_0) = 0$$

$$\Leftrightarrow \cos\psi = \frac{2}{3}\cos\theta_0$$

$$\Leftrightarrow \quad \psi = a\cos\left(\frac{2}{3}\cos\theta_0\right)$$

$\boxed{3}$



$\boxed{3/1}$ Syst : $\{M\}$

Réf : Labo Galiléen

Forces : $\vec{P}$, $\vec{T}$