- $\vec{F}$ est centrale conservative $\Leftrightarrow \vec{F} = F(r)\frac{\vec{OM}}{\|\vec{OM}\|} = -\frac{dE_p}{dr}$

$$= \frac{K}{r^2}\underbrace{\vec{u_r}}_{\frac{\vec{OM}}{\|\vec{OM}\|}} \Rightarrow E_p = \frac{K}{r} + \text{const}$$

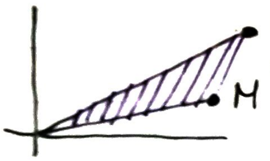
- Principe de conservation

$\begin{cases} M \text{ soumis à juste } \vec{F} = \frac{K}{r^2}\vec{u_r} \\ \mathcal{R} \text{ est un référentiel Galiléen} \end{cases}$

$$\Rightarrow \begin{cases} \dot{\vec{L_O}} = \vec{\mathcal{M}_O}\left(\frac{K}{r^2}\vec{u_r}\right) = \vec{OM} \wedge \frac{K}{r^2}\vec{u_r} = \vec{0} \Rightarrow \vec{L_O} = \vec{\text{const}} \\ \text{Mouvement de } M = \text{plan dans } \begin{cases} \perp \vec{L_O} \\ \ni O \end{cases} \\ \vec{OM}(0) \,//\, \vec{v}(0) \Rightarrow \vec{L_O} = \vec{0} \Rightarrow \text{mouvement rectiligne} \end{cases}$$

- Constante des aires $C := r^2\dot{\theta}^2 \wedge \begin{cases} C \overset{\bullet}{=} 0 \\ r^2 \overset{\bullet}{\neq} 0 \\ \dot{\theta}^2 \overset{\bullet}{\neq} 0 \end{cases}$ dérivée égale à

- 2e loi de Kepler



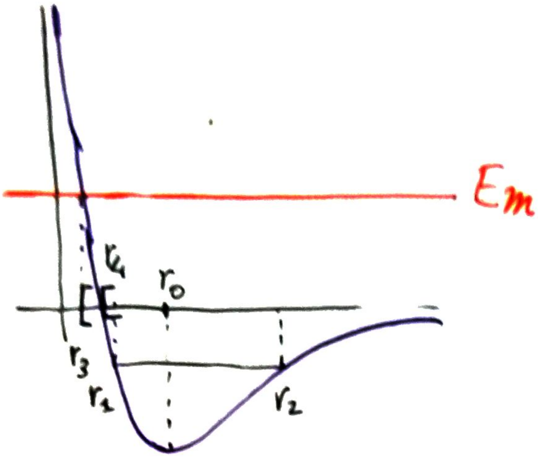
$d\mathcal{A}$ l'aire élémentaire

$$\begin{aligned} d\mathcal{A} &= \frac{1}{2}\|\vec{OM} \wedge d\vec{OM}\| \\ &= \frac{1}{2}\|r\vec{u_r} \wedge (dr\vec{u_r} + rd\theta\vec{u_\theta})\| \\ &= \frac{1}{2}\|r^2 d\theta \vec{u_z}\| \\ &= \frac{1}{2}r^2 d\theta \end{aligned}$$

$$\Leftrightarrow \underbrace{\dot{\mathcal{A}}}_{\text{vitesse aréolaire}} = \frac{1}{2}r^2\dot{\theta} = \frac{C}{2} = \text{const}$$

- TEM $E_m \overset{\bullet}{=} 0 \Rightarrow E_m := \underbrace{\frac{1}{2}m\dot{r}^2}_{E_{c_r}} + \underbrace{\frac{1}{2}m\frac{C^2}{r^2} + \frac{K}{r}}_{E_{p_{eff}}}$

Trajectoire & Ep_{eff}

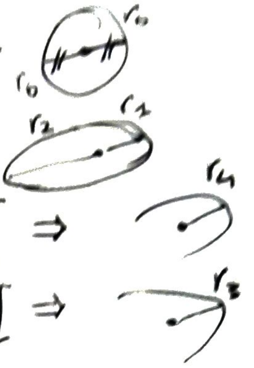


r / Ep_{eff} est min $\Rightarrow$

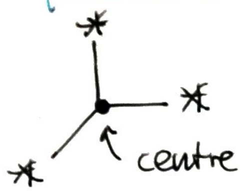
$r \in [r_1, r_2] \Rightarrow$

$r \in [r / Ep_{eff}(r) = 0, +\infty[\Rightarrow$ parabole

$r \in [r / Ep_{eff}(r) = E_m, +\infty[\Rightarrow$ hyperbole



Référentiels



$\begin{cases} \text{Terre} \Rightarrow \text{géocentrique} \\ \text{Soleil} \Rightarrow \text{héliocentrique} = \text{kepler} \\ \text{centre masse syst solaire} \Rightarrow \text{de Copernic} \end{cases}$

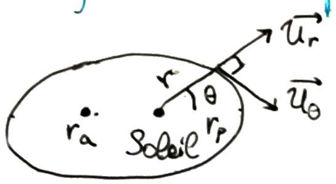
] Sur 99 jours, dépl $\approx$ TRU $\Rightarrow$ Galil

] $\approx$ Copernic (masses $\approx$) $\Rightarrow$ Galil

] Galil. absolu

⚠ Réf Terrestre $\neq$ géocentrique

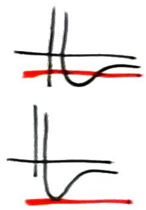
Trajectoires elliptiques

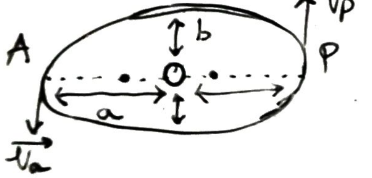


$$r(\theta) = \frac{p \text{ ``paramètre''}}{1 + (e \text{ ``excentricité''}) \cdot \cos\theta}$$

$\theta = 0 \Rightarrow$ "périhélie" $r_{min} := \dfrac{1}{p+e}$

$\theta = \dfrac{\pi}{2} \Rightarrow$ "aphélie" $r_{max} := \dfrac{1}{p-e}$





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$AP = 2(a \text{ ``demi grand-axe''}) = r_a + r_p = \frac{2p}{1+e^2}$$

en A et P : $E_m = Ep_{eff} \Rightarrow \frac{1}{2} m \dot{r}^2 = 0 \Rightarrow \dot{r_a} + \dot{r_p} \Rightarrow \vec{V_A}, \vec{V_P} \not\parallel \vec{u_r}$

$\Rightarrow \vec{V_A}, \vec{V_P} \parallel \vec{u_\theta}$

$$\overrightarrow{L_{Soleil}} = \overrightarrow{SoleilM} \wedge m\vec{v} = r\vec{u_r} \wedge m\underbrace{r\dot{\theta}\vec{u_\theta}}_{\vec{v}} = mr^2\dot{\theta}\vec{u_z} = mrv\vec{u_z} = \overrightarrow{const}$$

$$\|\overrightarrow{L_{Soleil}}\| = const = mrv \Rightarrow \cancel{m}r_a v_a = \cancel{m}r_p v_p \Rightarrow \begin{cases} \vec{v_a}, \vec{v_p} \,//\, \vec{u_\theta} \\ v_a < v_p \end{cases}$$

En A et P:

$$E_m = Ep_{eff}(r) = \frac{1}{2} m \frac{C^2}{r^2} \frac{\overbrace{-GM_s m}^{K}}{r}$$

r_a et r_p sont racines de $E_m r^2 + GM_s m r - \frac{1}{2} m \frac{C^2}{r^2}$

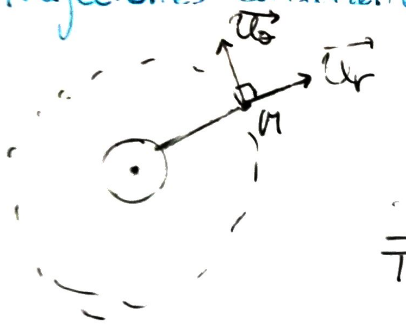
$$\Delta > 0 \Rightarrow r_{1,2} = -\frac{GM_s m \pm \sqrt{\Delta}}{2E_m}$$

$$\Rightarrow r_1 + r_2 = r_a + r_p = 2a = -G\frac{M_s m}{E_m}$$

$$\Rightarrow E_m = -G\frac{M_s m}{2a} = const < 0$$

$$= \frac{K}{2a}$$

- Trajectoires circulaires



Syst: {M} ; Réf: Géo gal. ; Forces:

$\cdot\ \vec{F} = -G\frac{M_T m}{(R_T+h)^2}\vec{u_r}$

$$\ddot{\overrightarrow{TR}} = \frac{d^2}{dt^2}(r\vec{u_r}) = \frac{d}{dt}(\cancel{\dot{r}\vec{u_r}} + r\dot{\theta}\vec{u_\theta})$$
$$= r\ddot{\theta}\vec{u_\theta} - r\dot{\theta}^2\vec{u_r}$$

2e loi N proj.

$$\begin{array}{ll} /\vec{u_r} & \cancel{r}\dot{\theta}^2\cancel{m} = G\frac{M_T\cancel{m}}{(R_T+h)^2} \Rightarrow \dot{\theta} = \sqrt{\frac{GM_T}{r^3}} \Rightarrow \omega := r\dot{\theta} = \sqrt{\frac{GM_T}{r}} \\ /\vec{u_\theta} & m\ddot{\theta} = 0 \Rightarrow \dot{\theta} = const \end{array}$$

à $r = R_T$: $\vec{F} = -G\frac{M_T m}{R_T^2}\vec{u_r} = m\vec{g_0} = -mg_0\underbrace{\vec{u_z}}_{\vec{u_r}} \Rightarrow g_0 = G\frac{M_T}{R_T^2} \Rightarrow \omega = \sqrt{\frac{g_0 R_T^2}{r}}$

◦ 3^e loi de Kepler

$$\frac{T^2}{r^3} = \frac{\left(\frac{2\pi}{\omega}\right)^2}{r^3} = \frac{\left(\frac{2\pi r}{\dot\theta}\right)^2}{r^3} = \frac{\frac{4\pi r^2}{\dot\theta^2}}{r^3} = \frac{\frac{4\pi r^2}{\frac{GM_T}{r}}}{r^3} = \frac{\frac{4\pi r^3}{GM_T}}{\cancel{r^3}} = \frac{4\pi}{GM_T} \doteq O$$

Pour une ellipse $\frac{T^2}{a^3} = \frac{4\pi}{GM_T}$

$$E_m = E_p + E_c = \frac{1}{2}m v^2 - G\frac{M_T m}{r}$$

$$= \frac{1}{2}m\sqrt{G\frac{M_T}{r}}^2 - G\frac{M_T m}{r} = G\frac{M_T m}{2r} - G\frac{M_T m}{r} = G\frac{M_T m}{r}\left(\frac{1}{2}-1\right)$$

$$= -G\frac{M_T m}{2r} = \text{const} < 0 = -E_c$$