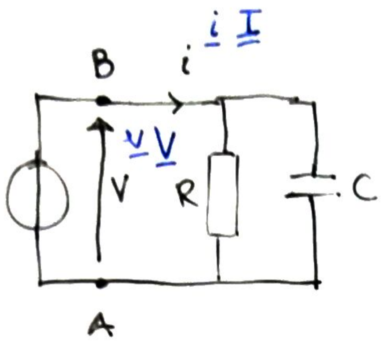
RSF, exercices

1/1/a



$v = V_m \cos \omega t$

$i = I_m \cos(\omega t + \varphi)$

$$\underline{Z}_R = R \qquad \underline{Z}_C = \frac{1}{jC\omega}$$

$$\underline{Z} = \frac{\underline{Z}_R \underline{Z}_C}{\underline{Z}_R + \underline{Z}_C} = \frac{\frac{R}{jC\omega}}{R + \frac{1}{jC\omega}} = \frac{R}{1 + jRC\omega}$$

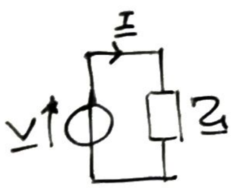
$$|\underline{Z}| = \frac{R}{\sqrt{1 + R^2C^2\omega^2}}$$

1/1/b

$$v = V_m \cos \omega t \Rightarrow \begin{cases} \underline{v} = V_m e^{j\omega t} \\ \underline{V} = V_m \end{cases}$$

$$i = I_m \cos(\omega t + \varphi) \Rightarrow \begin{cases} \underline{i} = I_m e^{j(\omega t + \varphi)} \\ \underline{I} = I_m e^{j\varphi} \end{cases}$$

alors



$$\underline{I} = \frac{\underline{V}}{\underline{Z}}$$

$$I_m = |\underline{I}| = \frac{V_m}{|\underline{Z}|} = \frac{V_m \sqrt{1 + R^2C^2\omega^2}}{R}$$

$$I_{eff} = \frac{I_m}{\sqrt{2}} = \frac{\frac{V_m}{\sqrt{2}} \sqrt{1 + R^2C^2\omega^2}}{R} = \frac{V_{eff} \sqrt{1 + R^2C^2\omega^2}}{R}$$

Déphasage de i/v

$$\arg \underline{I} - \arg \underline{V} = \arg \frac{\underline{I}}{\underline{V}} = \arg \frac{1}{\underline{Z}} = \arg \underline{Y}$$

$$= \arg\left(\frac{1}{R} + jC\omega\right) \quad \left(\frac{1}{\underline{Z}_{éq}} = \sum_k \frac{1}{\underline{Z}_k}\right)$$

en parallèle

or $\arg \underline{I} - \arg \underline{V} = \varphi - 0 = \varphi$

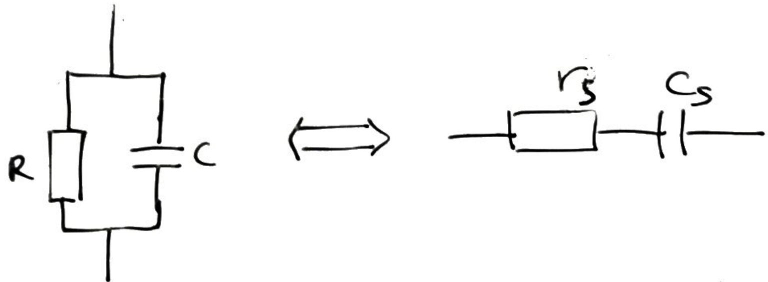
donc

$$\varphi = \arg\left(\frac{1}{R} + jC\omega\right)$$

$$\cos\varphi = \frac{\frac{1}{R}}{\left|\frac{1}{R} + jC\omega\right|} = \frac{1}{R\left|\frac{1}{R} + jC\omega\right|} = \frac{1}{R\sqrt{\frac{1}{R^2} + C^2\omega^2}} = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$

$$\tan\varphi = \frac{C\omega}{\frac{1}{R}} = RC\omega$$

1/1/c



$$\underline{Z} = \frac{R}{1 + jRC\omega} \;=\; \underline{Z}' = r_s + \frac{1}{jC_s\omega} = r_s - j\frac{1}{C_s\omega}$$

$$= \frac{R(1 - jRC\omega)}{(1 + jRC\omega)(1 - jRC\omega)}$$

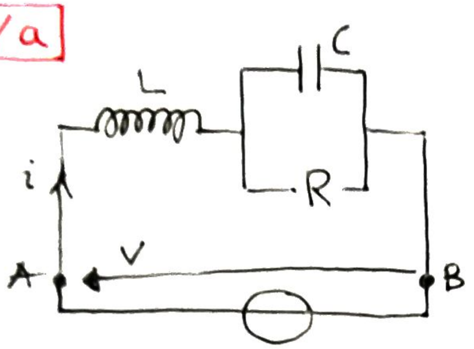
$\left((a+ib)(a-ib) = a^2 - (ib)^2 = a^2 + b^2 = |a+ib|^2 \right)$

$$= \frac{R(1 - jRC\omega)}{1 + R^2C^2\omega^2}$$

$$\underline{Z} = \underline{Z}' \Leftrightarrow \begin{cases} \frac{R}{1+R^2C^2\omega^2} = r_s \\ \frac{-R^2C\omega}{1+R^2C^2\omega^2} = -\frac{1}{C_s\omega} \Rightarrow \frac{1+R^2C^2\omega^2}{R^2C\omega^2} \end{cases}$$

RSF, Exos

2/a



$$\underline{Z}_{AB} = \underline{Z}_L + \frac{\underline{Z}_C \underline{Z}_R}{\underline{Z}_R + \underline{Z}_C}$$

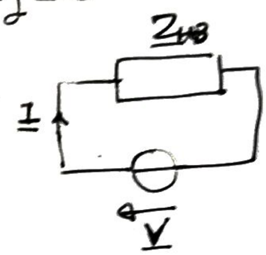
$$= jL\omega + \frac{\frac{R}{jC\omega}}{\frac{1}{jC\omega} + R}$$

$$= \frac{R}{1 + jRC\omega} + jL\omega$$

$$= \frac{R(1 - jRC\omega)}{1 + R^2C^2\omega^2} + jL\omega$$

2/b

$$\varphi = \arg \underline{I} - \arg \underline{V} = \arg \frac{\underline{I}}{\underline{V}}$$



$$= \arg \frac{1}{\underline{Z}_{AB}}$$

$$= -\arg \underline{Z}_{AB}$$

$$\tan \varphi = \tan(-\arg \underline{Z}_{AB}) = -\tan \arg \underline{Z}_{AB}$$

$$= -\frac{L\omega - \frac{R^2C\omega}{1+R^2C^2\omega^2}}{\frac{R}{1+R^2C^2\omega^2}} =: \text{truc}$$

$$\varphi = -\arctan(\text{truc})$$

$$\varphi = 0 \iff L\omega - \frac{R^2C\omega}{1+R^2C^2\omega^2} = 0$$

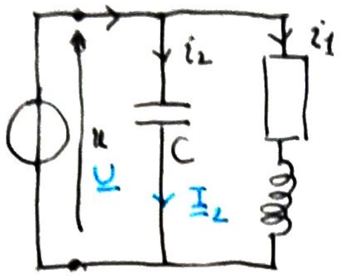
$$\iff L = \frac{R^2C}{1+R^2C^2\omega^2}$$

$$\iff L(1+R^2C^2\omega^2) = R^2C$$

$$\iff \omega^2 = \frac{R^2C}{LR^2C^2} - \frac{L}{LR^2C^2}$$

$$\iff \omega = \sqrt{\frac{1}{LC} - \frac{1}{R^2C^2}}$$

3



$$u(t) = U\sqrt{2}\cos(\omega t) : U_{max} = U\sqrt{2}$$

$$U_{eff} = \frac{U_{max}}{\sqrt{2}} = U$$

$$\underline{u} = U\sqrt{2}\, e^{j\omega t}$$

$$\underline{U} = U\sqrt{2}$$

$$\underline{I}_1 = \frac{\underline{U}}{R + jL\omega} \Rightarrow I_{1_m} = \frac{U_m}{\sqrt{R^2 + L^2\omega^2}} \Rightarrow I_{1eff} = \frac{U}{\sqrt{R^2 + L^2\omega^2}}$$

$$= 300 \text{ mA}$$

$$\underline{I}_2 = \frac{\underline{U}}{\underline{Z}_c} = \frac{\underline{U}}{\frac{1}{jC\omega}} = jC\omega\underline{U} \Rightarrow \begin{cases} \varphi_{i_2/u} = \frac{\pi}{2} \\ |\underline{I}_2| = I_{2_m} = C\omega U_m \end{cases}$$

$(\div\sqrt{2})$

$$I_{2eff} = C\omega U_{eff}$$

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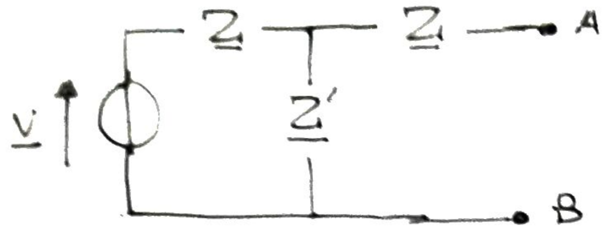
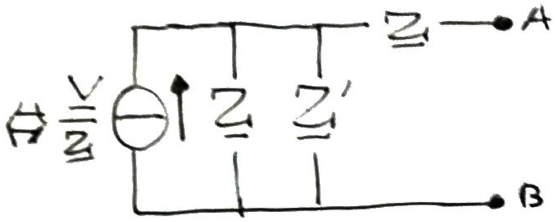
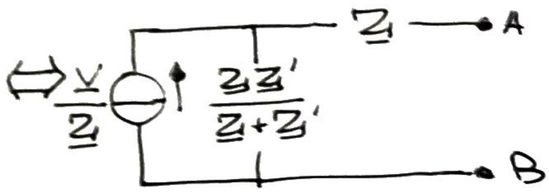
$$= 10^{-6} \cdot 50 \cdot 2\pi \cdot 100$$

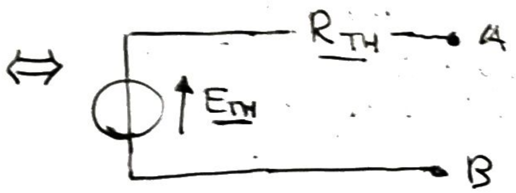
$$\approx 31 \text{ mA}$$

4/1



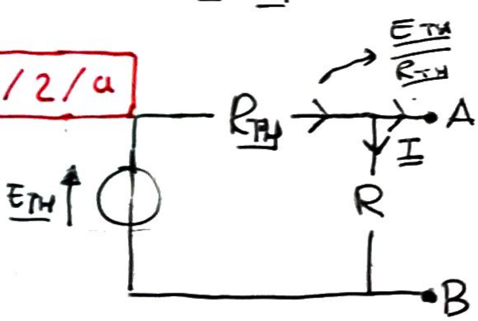






$$\underline{R}_{TH} = \frac{\underline{Z}\underline{Z}'}{\underline{Z}+\underline{Z}'} + \underline{Z} = \frac{2\underline{Z}\underline{Z}' + \underline{Z}^2}{\underline{Z}+\underline{Z}'} \qquad \underline{E}_{TH} = \underline{V}\,\underline{R}_{TH}$$

4/2/a



RSF, exos

$$\underline{E}_{TH} = \frac{\underline{V}\,\underline{Z}'}{\underline{Z}+\underline{Z}'} \qquad \underline{I} = \frac{\underline{E}_{TH}}{\underline{R}_{TH} + R}$$

$$\underline{R}_{TH} = \frac{\underline{Z}\,\underline{Z}'}{\underline{Z}+\underline{Z}'} + \underline{Z} \qquad = \frac{\frac{\underline{V}\,\underline{Z}'}{\underline{Z}+\underline{Z}'}}{\frac{\underline{Z}\,\underline{Z}'}{\underline{Z}+\underline{Z}'} + \underline{Z} + R}$$

$$= \frac{\underline{V}\,\underline{Z}'}{\underline{Z}\,\underline{Z}' + (\underline{Z}+\underline{Z}')(\underline{Z}+R)}$$

4/2/b $\underline{Z} = -\underline{Z}' \Leftarrow \underline{Z}+\underline{Z}' = 0$

4/2/c $\underline{Z}+\underline{Z}' = \frac{1}{jC\omega} + jL\omega$

$$\underline{Z}+\underline{Z}' = 0 \Leftrightarrow jL\omega = j\frac{1}{C\omega}$$

$$\Leftrightarrow \omega = \sqrt{\frac{L}{C}}$$

$$\underline{I} = \frac{\underline{V}\,\underline{Z}'}{\underline{Z}\,\underline{Z}'}$$

$$= \frac{\underline{V}}{\underline{Z}}$$

$$= j\underline{V}C\omega$$

$$|\underline{I}| = V_m C\omega = I_m$$

$$I_{eff} = \frac{I_m}{\sqrt{2}} = \frac{V_m}{\sqrt{2}}C\omega = V_{eff}C\omega$$

$$\arg \underline{I} = \arg \underline{V} + \arg(jC\omega)$$

$$= \arg \underline{V} + \frac{\pi}{2}$$

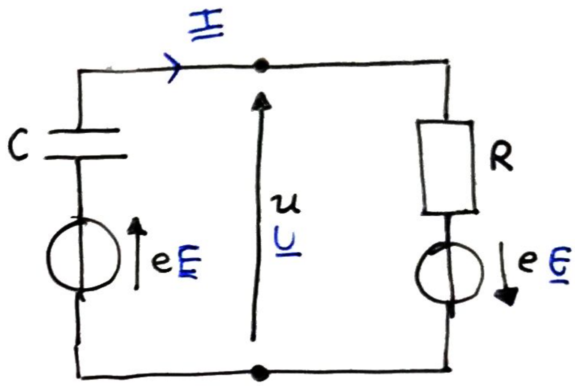
déphasage de $\underline{I}/\underline{V}$

$$\arg \underline{I} - \arg \underline{V} = \frac{\pi}{2}$$

5/1

$$e(t) = E\sqrt{2}\cos(\omega t)$$

$$u(t) = U\sqrt{2}\cos(\omega t + \varphi)$$



$$\begin{cases} \underline{e} = E\sqrt{2}e^{j\omega t} \\ \underline{E} = E\sqrt{2} \end{cases} \qquad \begin{cases} \underline{u} = U\sqrt{2}e^{j(\omega t+\varphi)} \\ \underline{U} = U\sqrt{2}e^{j\varphi} \end{cases}$$

$\begin{cases} E \\ U \end{cases}$ est la valeur efficace

Il faut trouver une relation entre $\underline{E}$ et $\underline{U}$.

$$\underline{I} = \frac{\underline{E} + \underline{E} - \frac{1}{jC\omega}}{R}$$

$$= \frac{2\underline{E}}{R + \frac{1}{jC\omega}}$$

$$\begin{cases} \underline{U} = \underline{E} - \frac{1}{jC\omega}\underline{I} = \underline{E} - \frac{2\underline{E}}{jRC\omega+1} = j\frac{\underline{E}RC\omega - \underline{E}}{jRC\omega + 1} = \frac{\underline{E}(jRC\omega - 1)}{jRC\omega + 1} \\ \underline{U} = (-\underline{E} + R\underline{I}) = -\underline{E} + \frac{2R\underline{E}}{R + \frac{1}{jC\omega}} = -\underline{E} + \frac{j2RC\omega\underline{E}}{jRC\omega + 1} \end{cases}$$

$$= \frac{jRC\omega\underline{E} - \underline{E}}{jRC\omega + 1}$$

$$= \frac{\underline{E}(jRC\omega - 1)}{jRC\omega + 1}$$

$\underline{E} = \sqrt{2}E \qquad \underline{U} = \sqrt{2}\,U e^{j\varphi}$

$$U\sqrt{2} = E\sqrt{2} \cdot \left| \frac{-1 + jRC\omega}{1 + jRC\omega} \right|$$

rem q : $\left| \frac{-a+ib}{a+ib} \right| = 1$

$$= E\sqrt{2} \cdot 1$$

$$\Leftrightarrow U = E$$

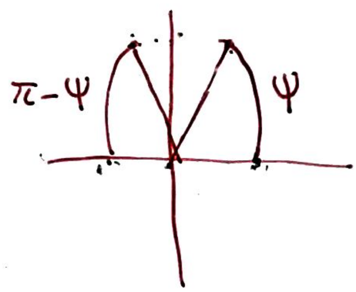
5/2

$$\arg \underline{U} = \arg \underline{E} + \arg \frac{jRC\omega - 1}{jRC\omega + 1}$$

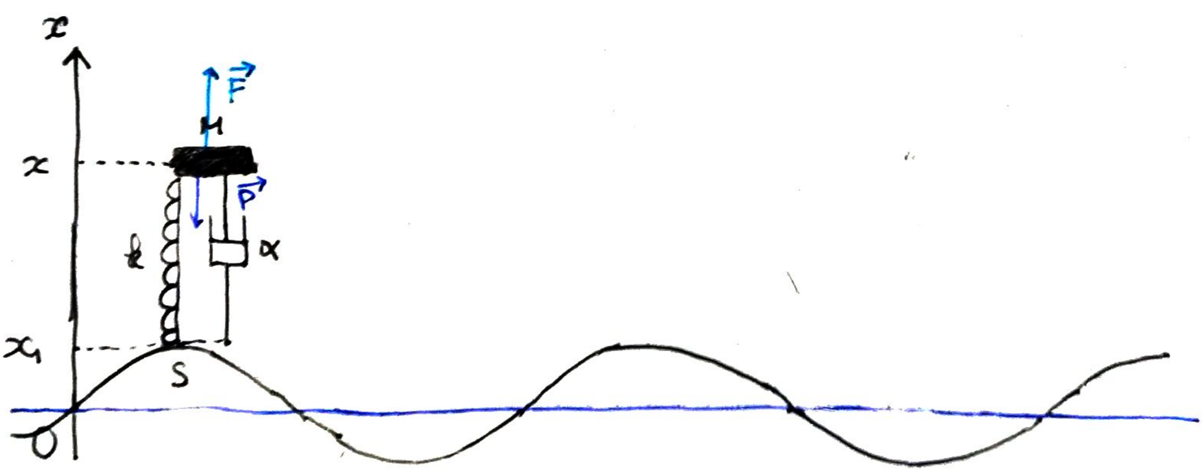
$$\varphi = 0 + \arg(jRC\omega - 1) - \arg(jRC\omega + 1)$$

Posons $\psi = \arg(1 + jRC\omega)$

$$\varphi = \arg(jRC\omega - 1) - \psi$$
$$= \pi - \psi - \psi$$
$$= \pi - 2\psi$$
$$= \pi - 2\,\mathrm{atan}(RC\omega)$$



6/1



$\vec{f_d} = -\alpha(v - v_1)\vec{u_x}$

Syst : $\{M\}$ Réf : labo, supp Galiléen.

Forces : $\vec{P}$, $\vec{F}$, ~~$\vec{f_d}$~~ $\vec{F}_{traction}$ pas de $\vec{R_n}$ car on étudie $\{M\}$

0

x_1, x const

$\Rightarrow v_1, v = 0$

$$m\vec{a} = \vec{P} + \vec{F} + \vec{F}_{traction}$$

On projette sur $(O\vec{x})$:

$$0 = -mg - k(x_e - l_0)$$

$$\Leftrightarrow x_e = -\frac{mg}{k} + l_0$$

6/2 $X = t \mapsto x(t) - x_e$

$$m\vec{a} = \vec{P} + \vec{F} + \vec{F}_{traction} + \vec{f_d}$$

On projette sur $(O\vec{x})$:

$$m\ddot{x} = -mg - k(x - x_1 - l_0) - \alpha(\dot{x} - \dot{x}_1)$$

$$X = x - x_e \Leftrightarrow x = X + x_e \Rightarrow \dot{X} = \dot{x} - 0 \Rightarrow \ddot{X} = \ddot{x}$$

$$m\ddot{x} = -\cancel{mg} - k(x - x_1 - \cancel{l_0} + \cancel{x_e} - x_e) - \alpha(\dot{x} - \dot{x}_1)$$

$$m\ddot{x} + \alpha\dot{x} + k(x - x_e) = kx_1 + \alpha v_1$$

$$m\ddot{X} + \alpha\dot{X} + kX = kx_1 + \alpha\dot{x}_1 = F(t)$$

6/3 $F(t)$ représente la force excitatrice provenant de l'ondulation de la route

6/3/a $F(t) = F_m \cos(\omega t)$

$$v(t) = v_m \cos(\omega t + \varphi) \qquad \underline{F} = F_m e^{j\omega t}$$
$$\underline{v} = v_m e^{j(\omega t+\varphi)} \qquad \underline{F}_t = F_m$$
$$\underline{V} = v_m e^{j\varphi}$$

$$\underline{H} := \frac{\underline{X}}{\underline{x}_1} \; ; \; \omega_0 := \sqrt{\frac{k}{m}} \; ; \; q := \frac{\alpha}{2\sqrt{mk}} \; ; \; p := \frac{\omega}{\omega_0}$$

$$m\ddot{\underline{X}} + \alpha\dot{\underline{X}} + k\underline{X} = \underline{F}$$

$$jm\omega\underline{v} + \frac{\alpha}{m}\underline{v} + \frac{k}{j\omega}\underline{v} = \underline{F}$$

$$\underline{v} = \frac{F}{\alpha + jm\omega + \frac{k}{j\omega}}$$

$$v_m = \frac{F_m}{\sqrt{\alpha^2 + (m\omega - \frac{k}{\omega})^2}}$$

6/3/b

$$m\ddot{\underline{X}} + \alpha\dot{\underline{X}} + k\underline{X} = k\underline{x}_1 + \alpha\dot{\underline{x}}_1$$

$$\Leftrightarrow -m\omega^2\underline{X} + j\alpha\omega\underline{X} + k\underline{X} = k\underline{x}_1 + j\alpha\omega\underline{x}_1$$

$$\Leftrightarrow \underline{X}(-m\omega^2 + j\alpha\omega + k) = \underline{x}_1(k + j\alpha\omega)$$

$$\Leftrightarrow \underline{H} = \frac{k + j\alpha\omega}{-m\omega^2 + j\alpha\omega + k}$$

$$= \frac{\omega_0^2 + j\alpha\frac{\omega}{m}}{-\omega^2 + j\alpha\frac{\omega}{m} + \omega_0^2} = \frac{1 + j\alpha\frac{\omega}{m\omega_0^2}}{-p^2 + j\alpha\frac{\omega}{m\omega_0^2} + 1} \quad \text{or} \quad j\frac{\alpha\omega}{m\omega_0^2} = \frac{2q\sqrt{mk}\,\omega}{m\omega_0^2}$$

($\sqrt{mk}/m = \omega_0$)

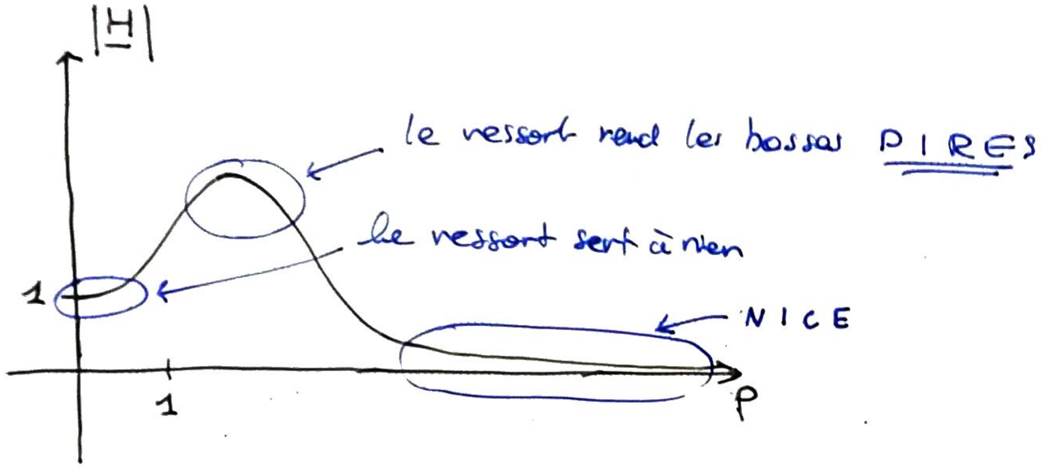
$$= j\frac{2q\omega_0\omega}{\omega_0^2}$$

$$= j2qp$$

On a :

$$\underline{H} = \frac{1+j2qp}{1-p^2+j2qp}$$

$$|\underline{H}| = \sqrt{\frac{1+4p^2q^2}{(1-p^2)^2+4p^2q^2}}$$



$|\underline{H}|$ est la réponse du point M par rapport à l'excitation

6/3/c

k grand $\Rightarrow \omega_0$ grand $\Rightarrow$ p et q petits

$\Rightarrow |\underline{H}|$ grand

$\Rightarrow$ ressort est KO

Il faut faire attention à ce que le ressort atténue le plus possible les oscillations