EXMSEA

$\boxed{1}$ Repère $(A, (\vec{AB}, \vec{AC})) = \left(\begin{pmatrix}1\\1\\1\end{pmatrix}, \left(\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right)\right)$

$$\vec{P} = \mathrm{Vect}\left(\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right)$$

$$= \left\{ \begin{pmatrix}x\\y\\-x\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix} \in \mathbb{R}^2 \right\}$$

$$P = \tau_{\begin{pmatrix}1\\1\\1\end{pmatrix}}(\vec{P})$$

$$= \left\{ \begin{pmatrix}x+1\\y+1\\-x+1\end{pmatrix}, \begin{pmatrix}x\\y\end{pmatrix} \in \mathbb{R}^2 \right\}$$

$$= \left\{ \begin{matrix}a+1\\b+1\\-a+1\end{matrix}, \begin{pmatrix}a\\b\end{pmatrix} \in \mathbb{R}^2 \right\}$$

$$= \left\{ \begin{pmatrix}x\\y\\z\end{pmatrix}, \exists a,b \in \mathbb{R}, \begin{cases} x = a+1 \\ y = b+1 \\ z = -a+1 \end{cases} \right\}$$

$$= \left\{ \begin{pmatrix}x\\y\\z\end{pmatrix}, \exists a \in \mathbb{R}, \begin{cases} x = a+1 \\ \\ z = -a+1 \end{cases} \right\}$$

car $b = y-1$, on remplace b par $y-1$

$$= \left\{ \begin{pmatrix}x\\y\\z\end{pmatrix}, z = -(x-1)+1 \right\}$$

car $a = x-1$, on remplace a par $x-1$.

$$= \left\{ \begin{pmatrix}x\\y\\z\end{pmatrix} \in \mathbb{R}^3, x+z = 2 \right\}$$

Meth 2 Un vecteur normal à $\vec{\mathcal{P}}$ (donc à $\mathcal{P}$) est $\vec{AB} \wedge \vec{AC}$

$$\vec{AB} \wedge \vec{AC} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

donc $\vec{\mathcal{P}} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \ x + z = 0 \right\}$

Et donc il existe d tel que

- $\mathcal{P} = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \ x + y = d \right\}$

$A \in \mathcal{P}$ donc $x_A + z_A = d$ ie $\boxed{d = 2}$

$\boxed{4/1}$

$$\Xi = \{y \in \mathcal{D}([0,1], \mathbb{R}),\ \forall x \in [0,1],\ y(x) + y'(x) + y(0) + y'(1) = 1\}$$

$$= \{y \in \mathcal{D}([0,1], \mathbb{R}),\ y + y' + y(0) + y'(1) = t \mapsto 1\}$$

C'est l'exemple fondamentale avec

$E = \mathcal{D}([0,1], \mathbb{R})$

$F = \mathcal{F}([0,1], \mathbb{R})$

$f = y \mapsto y + y' + y(0) + y'(1) \in \mathcal{L}(E, F)$ par lin. de $+, \cdot(\cdot\cdot), \cdot'$

$b = t \mapsto 1$

$\boxed{4/2}$

$\Xi \neq \emptyset$ car $t \mapsto \frac{1}{2} \in \Xi$

Donc $\Xi = \tau_{t \mapsto \frac{1}{2}}(\mathrm{Ker}\, f)$

Cherchons les y tels que $y \in \mathrm{Ker}\, f$

ie $y + y' + y(0) + y'(1) = 0$

ie $y + y' = \underbrace{-y(0) - y'(1)}_{C}$

Analyse On résout $y + y' = c$

$$S_H = K \cdot \exp \circ (-id) \quad \text{avec } K \in \mathbb{R}$$
$$= \mathbb{R}e^{-id}$$
$$S_p = t \mapsto c$$
$$S = \mathbb{R}e^{-id} + c$$

Synthèse Testons nos candidats.

$$\mathbb{R}e^{-id} + c \subset \operatorname{Ker} f$$

ie $\forall k \in \mathbb{R},\ ke^{-id} - ke^{-id} + k - ke^{-1} + 2c = 0$

ie $\forall k \in \mathbb{R},\ c = \dfrac{k(e^{-1} - 1)}{2}$

$$\operatorname{Ker} f = \left\{ ke^{-id} + \frac{k}{2}(e^{-1} - 1),\ k \in \mathbb{R} \right\}$$

$$\Xi = \operatorname{Ker} f + \frac{1}{2} = \left\{ ke^{-id} + \frac{k}{2}(e^{-1} - 1),\ k \in \mathbb{R} \right\}$$

Repère $\left(t \mapsto \frac{1}{2},\ \left(e^{-id} + \frac{1}{2}(e^{-1} - 1)\right)\right)$