Développements limités usuels en 0

V2

I Variantes sur la somme des termes d'une suite géométrique

$$\frac{1}{1-x} = 1+x+x^2+\ldots+x^n+o(x^n) = \left(\sum_{k=0}^{n} x^k\right)+o(x^n)$$

$$\frac{1}{1+x} = 1-x+x^2-\ldots+(-1)^n x^n+o(x^n) = \left(\sum_{k=0}^{n} (-1)^k x^k\right)+o(x^n)$$

$$\ln(1-x) = -x-\frac{x^2}{2}-\ldots-\frac{x^n}{n}+o(x^n) = \left(-\sum_{k=\mathbf{1}}^{n} \frac{x^k}{k}\right)+o(x^n)$$

$$\ln(1+x) = x-\frac{x^2}{2}+\ldots+\frac{(-1)^{n+1}x^n}{n}+o(x^n) = \left(\sum_{k=\mathbf{1}}^{n} \frac{(-1)^{k+1}x^k}{k}\right)+o(x^n)$$

$$\arctan(x) = x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots+\frac{(-1)^n x^{2n+1}}{2n+1}+o\left(x^{2n+1}\right) = \left(\sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{2k+1}\right)+o\left(x^{2n+1}\right)$$

II Variantes sur l'exponentielle

$$\mathrm{e}^x = 1+x+\frac{x^2}{2}+\ldots+\frac{x^n}{n!}+o(x^n) = \left(\sum_{k=0}^{n} \frac{x^k}{k!}\right)+o(x^n)$$

$$\mathrm{ch}(x) = 1+\frac{x^2}{2}+\frac{x^4}{24}+\ldots+\frac{x^{2n}}{(2n)!}+o\left(x^{2n}\right) = \left(\sum_{k=0}^{n} \frac{x^{2k}}{(2k)!}\right)+o\left(x^{2n}\right)$$

$$\mathrm{sh}(x) = x+\frac{x^3}{6}+\ldots+\frac{x^{2n+1}}{(2n+1)!}+o\left(x^{2n+1}\right) = \left(\sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!}\right)+o\left(x^{2n+1}\right)$$

$$\cos(x) = 1-\frac{x^2}{2}+\frac{x^4}{24}-\ldots+\frac{(-1)^n x^{2n}}{(2n)!}+o\left(x^{2n}\right) = \left(\sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}\right)+o\left(x^{2n}\right)$$

$$\sin(x) = x-\frac{x^3}{6}+\ldots+\frac{(-1)^n x^{2n+1}}{(2n+1)!}+o\left(x^{2n+1}\right) = \left(\sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}\right)+o\left(x^{2n+1}\right)$$

III Puissances de $(1+x)$ (<u>α doit être constant !</u>)

$$(1+x)^\alpha = 1+\alpha x+\frac{\alpha(\alpha-1)}{2}x^2+\frac{\alpha(\alpha-1)(\alpha-2)}{6}x^3+\ldots+\frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-n+1)}{n!}x^n+o(x^n)$$

$$= \left(\sum_{k=0}^{n} \binom{\alpha}{k} x^k\right)+o(x^n) \qquad \text{où l'on note } \binom{\alpha}{k} = \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-k+1)}{k!}$$