

On note $\mathbb{K} := \mathbb{R}$ ou \mathbb{C} .

1 Bolzano-Weierstraß

$$\forall u \in \mathbb{K}^{\mathbb{N}}, (\exists K \in \mathbb{R}_+, |u| \leq K) \implies (\exists \phi \in \mathbf{Inc}(\mathbb{N}, \mathbb{N}), \lim_{\infty} u \circ \phi \in \mathbb{R})$$

2 Bornes atteintes

$$\forall a < b \in \mathbb{R}, \forall f \in \mathcal{C}([a, b], \mathbb{R}), (\exists K \in \mathbb{R}_+, |f| \leq K) \wedge \left(\exists i, s \in D_f, \begin{cases} f(i) = \inf f \\ f(s) = \sup f \end{cases} \right)$$

2.1 Sur \mathbb{C}

$$\forall a < b \in \mathbb{R}, \forall f \in \mathcal{C}([a, b], \mathbb{C}), \exists K \in \mathbb{R}_+, |f| \leq K$$

”Atteindre ses bornes” n’a plus de sens dans \mathbb{C} .

2.2 Image d’un segment

$$\forall [a, b] \subset \mathbb{C}, \forall f \in \mathcal{C}, f^{\rightarrow}([a, b]) \subset \text{segments}(\mathbb{C})$$

3 Heine

$$\forall a < b \in \mathbb{R}, \mathcal{C}([a, b], \mathbb{R}) \subset \mathcal{UC}$$