EXM CCINP

$\boxed{81/2}$

$$\mathcal{F} = \mathrm{Vect}\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)$$

$$\Rightarrow \mathcal{F}^{\perp} = \mathrm{Vect}\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)^{\perp}$$

$$= \left\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right\}^{\perp}$$

$$= \left\{\begin{pmatrix} x & y \\ z & t \end{pmatrix} \in \mathcal{M}_2(\mathbb{R}),\ \begin{cases} \left\langle \begin{pmatrix} x & y \\ z & t \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\rangle = 0 \\ \left\langle \begin{pmatrix} x & y \\ z & t \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\rangle = 0 \end{cases}\right\}$$

$$= \left\{\begin{pmatrix} x & y \\ z & t \end{pmatrix} \in \mathcal{M}_2(\mathbb{R}),\ \begin{cases} x + t = 0 \\ y - z = 0 \end{cases}\right\}$$

$$= \left\{\begin{pmatrix} x & y \\ y & -x \end{pmatrix},\quad \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2\right\}$$

$$= \mathrm{Vect}\Bigg(\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\perp}\Bigg)$$

$$= \mathrm{Vect}\Bigg(\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\text{b.o.n.}}\Bigg)$$

$\boxed{81/3}$

$$p^{\perp}_{\mathcal{F}^{\perp}}\left(\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right) = \frac{1}{2}\underbrace{\left\langle\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}, \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right\rangle}_{0}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + \frac{1}{2}\underbrace{\left\langle\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}, \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right\rangle}_{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$\boxed{81/4}$

$$\begin{aligned} d(J, \mathcal{F}) &= d(J, p^{\perp}_{\mathcal{F}}(J)) \\ &= \| J - p^{\perp}_{\mathcal{F}}(J) \| \\ &= \| p^{\perp}_{\mathcal{F}^{\perp}}(J) \| \\ &= \left\| \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \right\| \\ &= \sqrt{2} \end{aligned}$$