EXMCCINP

$\boxed{80/1}$

- sym
- bilin.

ok.

- def-pos:

Si $\int_0^{2\pi} f^2 \cdot g = 0$ $\begin{cases} f^2 \geqslant 0 \\ f^2 \in \mathcal{C} \end{cases}$

On a

F tq $F' = f$

$0 = \int_0^{2\pi} f^2 = F(b) - F(a) \Rightarrow F(b) = F(a)$

$F' \geqslant 0$ donc $f \in \mathcal{L}([0, 2\pi], \mathbb{R})$

donc $F \in \mathcal{C}([0, 2\pi], \mathbb{R})$

$F' = 0$ sur $[0, 2\pi]$ $f^2 = 0$ donc $f = 0$

Or $f \in \mathcal{O}_{2\pi}$ donc $F' = 0$ sur $\mathbb{R}$

$\boxed{80/2}$

$$\|\cos \circ (2id)\| = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \cos 2t \cos 2t \, dt}$$

$$= \sqrt{\frac{1}{4\pi}\int_0^{4\pi} \cos(u)^2 \, du} \qquad u := 2t$$

$$= \sqrt{\frac{1}{8\pi}\Big[u + \cos u \sin u\Big]_{u=0}^{4\pi}} = \frac{1}{\sqrt{2}}$$

et $\|\cos\| = \frac{\sqrt{\pi}}{\sqrt{2\pi}} = \frac{1}{\sqrt{2}}$

$$\langle \cos, \cos\circ(2id)\rangle = \frac{1}{2\pi}\int_0^{2\pi} \cos \cdot \cos\circ(2id)$$

$$= \frac{1}{4\pi}\int_0^{2\pi} \cos\circ(3id) + \cos$$

$$= \frac{1}{12\pi}\int_0^{6\pi} \cos u \, du + \frac{1}{4\pi}\int_0^{2\pi} \cos \qquad u = 3id$$

$$= 0$$

On a une b.o.n. de F :

$$\left(\sqrt{2}\cos, \sqrt{2}\cos\circ(2id)\right)$$

$$p_F^{\perp}(u) = 2\langle u, \cos\rangle \cos + 2\langle u, \cos\circ 2id\rangle \cos\circ 2id$$

$$= \frac{1}{2\pi}\int_{\pi}^{-\pi} \sin^2 \cos$$

$$= \frac{1}{2\pi}\int_{\pi}^{-\pi} (1-\cos^2)\cos$$

$$= \underbrace{\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos}_{0} - \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos^3$$

$$= \frac{-1}{2\pi}\int_{-\pi}^{\pi} \cos^3$$

$$= \frac{1}{2\pi}\int_{\pi}^{-\pi} \frac{(e^{it}+e^{-it})^3}{8}\,dt$$

$$= \underbrace{\frac{1}{2\pi}\int_{\pi}^{-\pi} 4\cos\circ 3id}_{0} + \underbrace{\frac{1}{2\pi}\int_{\pi}^{-\pi} \frac{3}{8}\cos}_{0}$$

$$= 0$$

Donc $p_F^{\perp}(\sin^2) = 2\langle \sin^2, \cos\circ 2id\rangle \cos\circ 2id$

$$= 2\cos\circ(2id)\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin^2 \cos$$

$$= \frac{\cos\circ 2id}{\pi}\int_{-\pi}^{\pi} \frac{(e^{it}-e^{-it})^2}{-4} \cdot \frac{(e^{it}+e^{-it})}{2}\,dt$$

$$= \frac{-\cos\circ 2id}{\pi}\int_{-\pi}^{\pi} e^{2it}+e^{-2it}$$

$\vdots$

$$= -\frac{\cos\circ 2id}{\pi}\,\frac{\pi}{2}$$

$$= -\frac{1}{2}\cos\circ 2id$$

Meth 2 Astuce qui marche bien

$$\sin^2 = \frac{1-\cos}{2}$$

$$p_F^\perp(\sin^2) = p_F^\perp\Big(\underbrace{\tfrac{1}{2}}_{\in F^\perp}\Big) - \frac{1}{2}p_F^\perp\Big(\underbrace{\cos \circ 2id}_{F}\Big) \quad \text{par linéarité}$$

$$= 0 - \frac{1}{2}\cos \circ 2id$$