

CCINP 43

Seit $x_0 \in \mathbb{R}$. $\begin{cases} u_0 := x_0 \\ \forall n \in \mathbb{N}, u_{n+1} = \arctan u_n \end{cases}$

1.a.

1^{er} cas ($u_1 < u_0$):

$$\text{Factan } \in \mathcal{F} / (\mathbb{R}, \mathbb{R}) \Rightarrow \begin{cases} \arctan u_1 < \arctan u_0 \\ \text{i.e. } u_2 < u_1 \end{cases}$$

\vdots

$$\forall n \in \mathbb{N}, u_{n+1} < u_n$$

$$\Rightarrow u \in \mathcal{F}(\mathbb{N}, \mathbb{R})$$

2^e cas ($u_1 \geq u_0$):

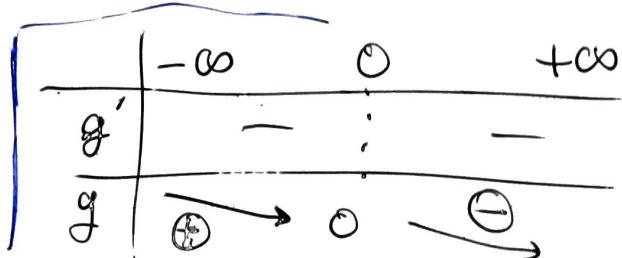
$$\text{idem} \Rightarrow u \in \mathcal{F}(\mathbb{N}, \mathbb{R}).$$

3^e cas ($u_1 = u_0$):

u constant ($\in \mathcal{C}(\mathcal{F} \cup \mathcal{F})$)

$$g := \arctan - \text{id}_{\mathbb{R}}$$

$$\begin{cases} g' = \frac{-(\text{id})^2}{1+(\text{id})^2} < 0 \Rightarrow g \in \mathcal{F} \\ g(0) = 0 \quad \text{tab var} \end{cases}$$



1.b.

$$\left| \begin{array}{l} f \in C(\mathbb{R}, \mathbb{R}) \\ \text{interv } \text{fermé} \\ x_0 \in \mathbb{R} \end{array} \right.$$

cas ($x_0 > 0$):

$u \in \mathbb{F}$ et $u \neq 0 \Rightarrow u \rightarrow 0$, seul pt fixe

cas ($x_0 = 0$): TLM

$$u = (0)_{n \in \mathbb{N}}$$

cas ($x_0 < 0$):

$u \in \mathbb{F}$ et $u \neq 0 \Rightarrow u \rightarrow 0$, seul pt fixe TLM

2. Soit $h \in C(\mathbb{R}, \mathbb{R})$ tq $h = h \circ \text{atan}$.

Soit $x \in \mathbb{R}$. Soit $\begin{cases} u_0 := x \\ \forall n \in \mathbb{N}, u_{n+1} := \text{atan} \circ u_n \end{cases}$

$$h(x) = \overbrace{h(u_0)}^{= h(u_1)} = h \circ \text{atan}(u_0)$$
$$= h(u_1)$$
$$= h \circ \text{atan} u_1$$
$$\vdots$$

$$\forall n \in \mathbb{N}, h(x) = h(u_n)$$

De plus, $h \circ u \xrightarrow{} h(0)$ car $h \in C$
donc $h(x) \xrightarrow{} h(0)$ par ! de lim

donc $h(x) = h(0)$

donc h constante.

Réciprocement & les fonctions convexes concaves.

Exemple : seules les fonctions convexe concave.