1. Soit $\frac{3X+7}{(X+1)^2} \in \mathbb{R}(X)$.

   Posons:

$$\frac{3X+7}{(X+1)^2} = \frac{A}{X+1} + \frac{B}{(X+1)^2}$$

   On a:

$$\begin{aligned} \frac{3X^2+7X}{(X+1)^2} &= \frac{AX}{X+1} + \frac{BX}{(X+1)^2} \\ X \to \infty \implies \frac{3}{1} &= \frac{A}{1} + 0 \\ &\implies \boxed{\text{A=3}} \end{aligned}$$

   Et:

$$\begin{aligned} X = 0 \implies \frac{7}{1} &= \frac{3}{1} + \frac{B}{1} \\ &\implies \boxed{\text{B=4}} \end{aligned}$$

   D'où $\boxed{f = x \mapsto \frac{3}{x+1} + \frac{4}{(x+1)^2}}$.

2. On a

$$f = 3 \cdot \left(x \mapsto \frac{1}{1+x}\right) + 4 \cdot \left(x \mapsto \frac{1}{1+x}\right)^2$$

   Or $x \to 0$, donc on effectue le $\mathrm{DL}_n(0)$ de $\frac{1}{1+\mathrm{id}}$, puis, par linéarité:

$$\begin{aligned} f(x) &= 3\sum_{k=0}^{n}(-1)^k x^k + 4\left(\sum_{k=0}^{n}(-1)^k x^k\right)^2 + o(x^n) \\ &= \end{aligned}$$

3. Particularisons en $n = 3$.