1

1.1

$f := \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix} = \begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix}$ où $\lambda \in \mathbb{R}$

$$\textit{On identifie} \begin{cases} a &= \lambda \\ b &= 0 \\ c &= 0 \\ d &= \lambda \end{cases}$$

Notons $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \lambda I_2$

1.2

f est la symmétrie d'axe Δ

1.3

Notons $z = x + iy$ On sait que $z' = x' + iy' = e^{i\theta} z$

$$\begin{aligned} f\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} \Re(e^{i\theta}(x+iy)) \\ \Im(e^{i\theta}(x+iy)) \end{pmatrix} \\ &= \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix} \\ e^{i\theta}(x+iy) &= (\cos\theta + i\sin\theta)(x+iy) \\ &= x\cos\theta + ix\sin\theta - iy\cos\theta - y\sin\theta \\ ax+by &= x\cos\theta - y\sin\theta \\ cx+dy &= x\sin\theta + y\cos\theta \end{aligned}$$

$$\begin{cases} a &= \cos\theta \\ b &= -\sin\theta \\ c &= \sin\theta \\ d &= \cos\theta \end{cases} \text{ et } A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Meth 2 *cours* : $f : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

où

- $\begin{pmatrix} a \\ c \end{pmatrix} = f(e_1) = f\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- $\begin{pmatrix} b \\ d \end{pmatrix} = f(e_2) = f\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

1. f homotéthie de rapport λ
   - $f(e_1) = f\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$
   - $f(e_2) = f\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}$

2. f symmétrie d'axe Δ

   - $f(e_1) = f\begin{pmatrix}1\\0\end{pmatrix} = e_2 = \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a\\c\end{pmatrix}$
   - $f(e_2) = f\begin{pmatrix}0\\1\end{pmatrix} = e_1 = \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}b\\d\end{pmatrix}$

3. f rotation d'angle θ

   - $f(e_1) = \begin{pmatrix}\cos\theta\\\sin\theta\end{pmatrix} = \begin{pmatrix}a\\c\end{pmatrix}$
   - $f(e_2) = \begin{pmatrix}-\sin\theta\\\cos\theta\end{pmatrix} = \begin{pmatrix}b\\d\end{pmatrix}$