Sommes et identités remarquables, fiche

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \prod_{k=1}^{n} k = n!$$

$$\sum_{k=a}^{b} 1 = b-a+1 \qquad \prod_{k=a}^{b} 1 = 1$$

$$\sum_{k=a}^{b} \lambda u_k + \mu v_k = \lambda \sum_{k=a}^{b} (u_k) + \mu \sum_{k=a}^{b} v_k \qquad \prod_{k=a}^{b} \lambda u_k + \mu v_k = \lambda^{b-a} \prod_{k=a}^{b} u_k \cdot \mu^{b-a} \prod_{k=a}^{b} v_k$$

$$\begin{cases} \sum_{j=c}^{d} u_j = \sum_{i=a}^{b} u_{\phi^{-1}(j)} \\ \sum_{i=a}^{b} u_i = \sum_{j=c}^{d} u_{\phi(j)} \end{cases} \qquad \begin{cases} \prod_{j=c}^{d} u_j = \prod_{i=a}^{b} u_{\phi^{-1}(j)} \\ \prod_{i=a}^{b} u_i = \prod_{j=c}^{d} u_{\phi(j)} \end{cases}$$

$$\sum_{k=0}^{n} a_{k+1} - a_k = a_{n+1} - a_0 \qquad \prod_{k=0}^{n} \frac{a_{k+1}}{a_k} = \frac{a_{n+1}}{a_0}$$

Soit u une suite arith de raison r

$$\sum_{k=0}^{n} u_k = (n+1)\frac{u_0 + u_n}{2}$$

Soit u une suite géo de raison q

$$\sum_{k=0}^{n} u_k = \begin{cases} \frac{q^{n+1} - 1}{q - 1} u_0 & \text{si } q \neq 1 \\ (n+1) u_0 & \text{sinon} \end{cases}$$

Soit u une suite arith-géo d'itératrice $x \mapsto \alpha x + \beta$ avec $\alpha \neq 1$

$$u_n = \alpha^n u_0 \beta \frac{1 - \alpha^n}{1 - \alpha}$$

$$\binom{n}{n-k} = \binom{n}{k}$$

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

Les Fubiniketamère

$$\sum_{i=0}^{m}\sum_{j=0}^{n} f(i,j) = \sum_{j=0}^{n}\sum_{i=0}^{m} f(i,j)$$

$$\sum_{i=0}^{n}\sum_{j=0}^{m} u_i v_i = \sum_{i=0}^{n} u_i \sum_{j=0}^{m} v_i$$

$$\sum_{i=0}^{n}\sum_{j=0}^{i} f(i,j) = \sum_{j=0}^{n}\sum_{i=j}^{n} f(i,j)$$