Mtq, $\mathbb{U}_n = \{z^n \in \mathbb{C},\ z^n = 1\} = \{e^{\frac{2i\pi k}{n}},\ k \in [\![1, n-1]\!]\}$
$\forall n \in \mathbb{N}^*$

$\boxed{\supset}$

$$\left(e^{\frac{2i\pi k}{n}}\right)^n = e^{2i\pi k} = \left(e^{2i\pi}\right)^k = 1^k = 1$$

$\boxed{\subset}$ Soit z tq $z^n = 1$.

Posons $z = re^{i\theta}$ avec $r = |z|$ et $\theta = \arg z$.

$$z^n = 1 \Leftrightarrow r^n e^{i\theta n} = 1 \Leftrightarrow \begin{cases} r^n = 1 \\ \theta n \equiv 0\ [2\pi] \end{cases}$$

$$\Leftrightarrow \begin{cases} r = \sqrt[n]{1} = 1 \\ \theta \equiv 0\ \left[\frac{2\pi}{n}\right] \end{cases}$$

$$\Leftrightarrow \begin{cases} r = 1 \\ \exists k \in \mathbb{Z},\ \theta = \frac{2\pi k}{n} \end{cases}$$

$$\Leftrightarrow \exists k \in \mathbb{Z},\ z^n = e^{\frac{2i\pi k}{n}}$$

D'après le th de div euclid, pour $k \in \mathbb{Z}$:

$\exists (q, r) \in \mathbb{Z}^2,\ k = qn + r$ et $0 \leqslant r < n$

donc $r \in [\![1, n-1]\!]$.

$$e^{\frac{2i\pi(qn+r)}{n}} = e^{2i\pi q} e^{\frac{2i\pi r}{n}} = e^{\frac{2i\pi r}{n}}$$

or $r \in [\![1, n-1]\!]$, donc

$$\{e^{\frac{2i\pi k}{n}},\ k \in [\![1, n-1]\!]\} \supset \{z \in \mathbb{C},\ z^n = 1\}$$